Chapter 6

Momentum and Collisions

Momentum

- The linear momentum of an object of mass *m* moving with a velocity v is defined as the product of the mass and the velocity
  - $p = m v$
  - SI Units are kg m / s
  - Vector quantity, the direction of the momentum is the same as the velocity's

Momentum

- In its simplest sense, **momentum** is a measure of motion, or the inertia of motion.
- Newton never used the word **momentum**, but when he stated motion in his first law, he was really talking about **momentum**.

Momentum components

- $p_x = m v_x$ and $p_y = m v_y$
- Applies to two-dimensional motion

Impulse

- In order to *change* the momentum of an object, a force must be applied
- The time rate of change of momentum of an object is equal to the net force acting on it
  - $F_{net} = \frac{\Delta p}{\Delta t} = \frac{m(v_f - v_i)}{\Delta t} = ma$
  - Gives an alternative statement of Newton's second law

Impulse cont.

- When a single, constant force acts on the object
  - $\Delta p = F \Delta t$
  - $F\Delta t$ is defined as the *impulse*
  - Vector quantity, the direction is the same as the direction of the force

Impulse-Momentum Theorem

- The theorem states that the impulse acting on the object is equal to the change in momentum of the object
  - $F\Delta t = \Delta p$
- If the force is not constant, use the *average force* applied

Momentum and Impulse

- Just like energy and work, where we said work changes energy, we can say that *impulse is equal to any change in momentum*:

$$J = \Delta p$$

- Newton actually wrote his 2^{nd} Law in this form!

Momentum and Impulse

- Lets look at this, and substitute in our equations.
- The middle equation is the best form for us to use.

$$J = \Delta p$$

$$Ft = \Delta(mv)$$

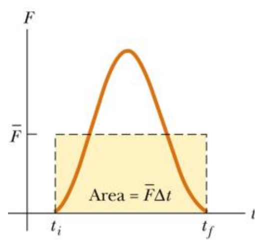
$$Ft = m\Delta v$$

$$F = \frac{m\Delta v}{t}$$

$$F = ma$$

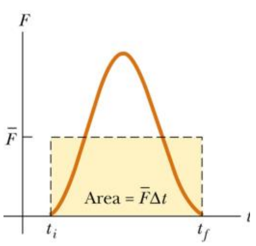
Force vs time graph

- The area under a force time curve is momentum.
- To calculate this you can break the shape into smaller figures.



Average Force in Impulse

- The average force can be thought of as the constant force that would give the same impulse to the object in the time interval as the actual time-varying force gives in the interval



Average Force cont.

- The impulse imparted by a force during the time interval Δt is equal to the area under the force-time graph from the beginning to the end of the time interval
- Or, to the average force multiplied by the time interval

Impulse Applied to Auto Collisions

- The most important factor is the collision time or the time it takes the person to come to a rest
  - This will reduce the chance of dying in a car crash
- Ways to increase the time
  - Seat belts
  - Air bags

Air Bags

- The air bag increases the time of the collision
- It will also absorb some of the energy from the body
- It will spread out the area of contact
  - decreases the pressure
  - helps prevent penetration wounds

Conservation of Momentum

- Momentum in an isolated system in which a collision occurs is conserved
  - A collision may be the result of physical contact between two objects
  - "Contact" may also arise from the electrostatic interactions of the electrons in the surface atoms of the bodies
  - An isolated system will have no external forces

Conservation of Momentum

- The principle of conservation of momentum states when no external forces act on a system consisting of two objects that collide with each other, the total momentum of the system before the collision is equal to the total momentum of the system after the collision

Conservation of Momentum, cont.

- Mathematically:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

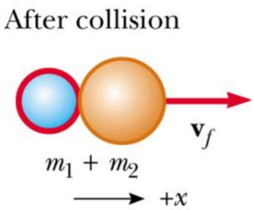
  - Momentum is conserved for the *system* of objects
  - The system includes all the objects interacting with each other
  - Assumes only internal forces are acting during the collision
  - Can be generalized to any number of objects

General Form of Conservation of Momentum

- The total momentum of an isolated system of objects is conserved regardless of the nature of the forces between the objects

Perfectly Inelastic Collisions

- When two objects stick together after the collision, they have undergone a inelastic collision
- Conservation of momentum becomes



$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

More Types of inelastic collisions

- inelastic collision (Not Perfect)
  - Objects bounce off of each other
  - The objects deform a little during the collision
  - Momentum is conversed but kinetic energy is not.
- Conservation of Momentum is followed

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

More about Collisions

- Momentum is conserved in any collision
- Inelastic collisions
  - Objects deform
  - They can stick together or bounce of each other
  - Kinetic energy is not conserved
    - Some of the kinetic energy is converted into other types of energy such as heat, sound, work to permanently deform an object
  - Deformation occurs ← It doesn't bounce back to its original shape
    - The greater the deformation the greater the KE loss

- Everyday collisions
  - Most collisions are inelastic

Some General Notes About Collisions

- Momentum is a vector quantity
  - Direction is important
  - Be sure to have the correct signs

More Types of Collisions

- Elastic collision (Sometimes called Perfect Elastic Collisions)
  - both momentum and kinetic energy are conserved
- Conservation of Momentum is followed

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

- Conservation of Energy is followed

$$\frac{1}{2} m_1 v^2{}_{1i} + \frac{1}{2} m_2 v^2{}_{2i} = \frac{1}{2} m_1 v^2{}_{1f} + \frac{1}{2} m_2 v^2{}_{2f}$$

Elastic Collisions

- Objects shapes remain the same ← It bounces back to its original shape
- Typically have two unknowns
  - Solve the equations simultaneously

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

Elastic Collisions, cont.

- A simpler equation can be used in place of the KE equation to see if energy is conserved.

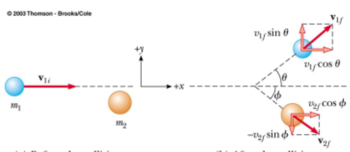
$$v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$$

- LOOK PAGE 173 to see the derivation of this equation

Glancing Collisions

- For a general collision of two objects in three-dimensional space, the conservation of momentum principle implies that the *total momentum of the system in each direction is conserved*
  - $m_1v_{1ix} + m_2v_{2ix} = m_1v_{1fx} + m_2v_{2fx}$ and
    $m_1v_{1iy} + m_2v_{2iy} = m_1v_{1fy} + m_2v_{2fy}$
  - Use subscripts for identifying the object, initial and final, and components

Glancing Collisions

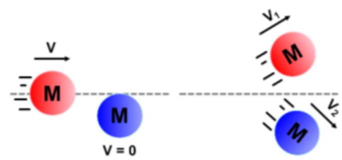


(a) Before the collision (b) After the collision

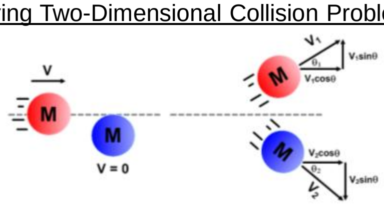
- The "after" velocities have x and y components
- Momentum is conserved in the x direction and in the y direction
- Apply separately to each direction

Solving Two-Dimensional Collision Problems:

A mass M collides with an identical mass M that is initially at rest. The first mass strikes slightly off-center of the second mass, causing both masses to be deflected at an angle to the x-axis. Determine the velocity of each mass after the collision.



Solving Two-Dimensional Collision Problems:



STEP 1: The X-Direction

$m_1v_1 + m_2v_2 = m_1v_1 + m_2v_2$

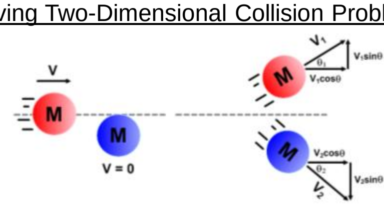
$(M)(V) + (M)(0) = (M)(v_1\cos\theta_1) + (M)(v_2\cos\theta_2)$

$MV = Mv_1\cos\theta_1 + Mv_2\cos\theta_2$

$\cancel{M}(V) = \cancel{M}(v_1\cos\theta_1 + v_2\cos\theta_2)$

$V = v_1\cos\theta_1 + v_2\cos\theta_2$

Solving Two-Dimensional Collision Problems:



STEP 2: The Y-Direction

$m_1v_1 + m_2v_2 = m_1v_1 + m_2v_2$

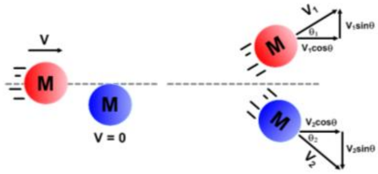
$(M)(0) + (M)(0) = (M)(v_1\sin\theta_1) + (M)(v_2\sin\theta_2)$

$0 = \cancel{M}v_1\sin\theta_1 + \cancel{M}v_2\sin\theta_2$

$0 = v_1\sin\theta_1 + v_2\sin\theta_2$

$v_1\sin\theta_1 = - v_2\sin\theta_2$

Solving Two-Dimensional Collision Problems:



STEP 3: Substitute X and Y Equations Into Each Other

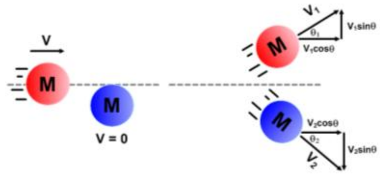
X: $V = v_1\cos\theta_1 + v_2\cos\theta_2$

Y: $v_1\sin\theta_1 = - v_2\sin\theta_2$

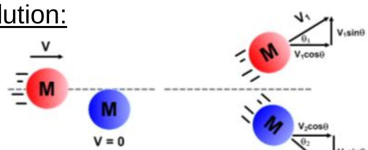
STEP 4: Insert given values and solve for the unknown variable

Example:

A 2.0-kg mass traveling at 5.0 m/s strikes another 2.0-kg mass that is initially at rest as shown below. After the collision, the first 2.0-kg travels at 2.0 m/s at an angle of 28º. What is the speed and direction of the second mass after the collision?



Solution:



X: $V = v_1\cos\theta_1 + v_2\cos\theta_2$

Y: $v_1\sin\theta_1 = - v_2\sin\theta_2$

X: $5 = 2\cos 28 + v_2\cos\theta_2$

Y: $2\sin 28 = - v_2\sin\theta_2$

X: $3.23 = v_2\cos\theta_2$

Y: $-0.94 = v_2\sin\theta_2$

X: $v_2 = 3.23/\cos\theta_2$

Y: $-0.94 = (3.23/\cos\theta_2)\sin\theta_2$

Y: $-0.94 = (3.23/\cos\theta_2)\sin\theta_2$

$-0.29 = \sin\theta_2/\cos\theta_2$

$-0.29 = \tan\theta_2$

$\boldsymbol{\theta_2 = 16^o}$

X: $3.23 = v_2\cos\theta_2$

$3.23 = v_2\cos(16)$

$v_2 = 3.23/\cos(16)$

$\mathbf{v_2 = 3.4\ m/s}$

Momentum of Split Particles

- An objects momentum is always conserved
- Objects that split apart have the summation of their particles momentum add up to the original momentum before the split

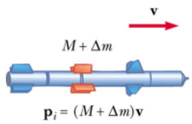
Rocket Propulsion

- The operation of a rocket depends on the law of conservation of momentum as applied to a system, where the system is the rocket plus its ejected fuel
  - This is different than propulsion on the earth where two objects exert forces on each other
    - road on car
    - train on track

Rocket Propulsion, cont.

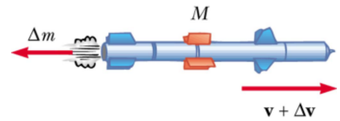
- The rocket is accelerated as a result of the thrust of the exhaust gases
- This represents the inverse of an inelastic collision
  - Momentum is conserved
  - Kinetic Energy is increased (at the expense of the stored energy of the rocket fuel)

Rocket Propulsion



- The initial mass of the rocket is M + Δm
  - M is the mass of the rocket
  - m is the mass of the fuel
- The initial velocity of the rocket is v

Rocket Propulsion



- The rocket's mass is M
- The mass of the fuel, Δm, has been ejected
- The rocket's speed has increased to v + Δv

Rocket Propulsion, final

- The basic equation for rocket propulsion is:

$$v_f - v_i = v_e \ln\left(\frac{M_i}{M_f}\right)$$

  - M_i is the initial mass of the rocket plus fuel
  - M_f is the final mass of the rocket plus any remaining fuel
  - The speed of the rocket is proportional to the exhaust speed

Thrust of a Rocket

- The thrust is the force exerted on the rocket by the ejected exhaust gases
- The instantaneous thrust is given by

$$Ma = M\frac{\Delta v}{\Delta t} = \left|v_e \frac{\Delta M}{\Delta t}\right|$$

  - The thrust increases as the exhaust speed increases and as the burn rate ($\Delta M/\Delta t$) increases

Show Mechanical Universe 15
Conservation of Momentum

$$Ma = M\frac{\Delta v}{\Delta t} = \left|v_e \frac{\Delta M}{\Delta t}\right|$$

Newton's Cradle