Chapter 6

Centripetal Force
and
The Law of Gravity

Radians

- In a circle the ratio of the arc length to the radius remains constant even if a circle in increased in size.
- This ratio is called the *radian*

$$\theta = \frac{s}{r}$$

Radians

- Based on the equation for the circumference of the circle

$$Circumference = 2\pi r$$

- So a complete circle has

$$\theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi \_ radians$$

More About Radians

- Comparing degrees and radians

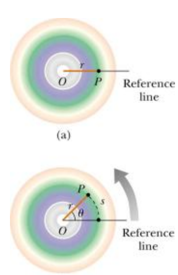
$$1\,\text{rad} = \frac{360°}{2\pi} = 57.3°$$

- Converting from degrees to radians

$$\theta\,[\text{rad}] = \frac{\pi}{180°}\,\theta\,[\text{degrees}]$$

Angular Displacement

- Axis of rotation is the center of the disk
- Need a fixed reference line
- During time t, the reference line moves through angle θ



Angular Displacement, cont.

- Every point on the object undergoes circular motion about the point O
- Angles generally need to be measured in *radians*

$$\theta = \frac{s}{r}$$

- s is the length of arc and r is the radius

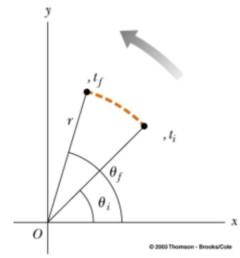
More about Angular Displacement

- The *angular displacement* is defined as the angle the object rotates through during some time interval
- Every point on the disc undergoes the same angular displacement in any given time interval

Angular Velocity

- The average angular velocity, ω, of a rotating rigid object is the ratio of the angular displacement to the time interval

$$\bar{\omega} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$



More about Angular Velocity

- The *instantaneous* angular velocity is defined as the limit of the average speed as the time interval approaches zero
- Units of angular velocity are radians/sec
  - rad/s
- Speed will be positive if θ is increasing (counterclockwise)
- Speed will be negative if θ is decreasing (clockwise)

Angular Acceleration

- The average angular acceleration, α, of an object is defined as the ratio of the change in the angular velocity to the time it takes for the object to undergo the change:

$$\bar{\alpha} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

More About Angular Acceleration

- Units of angular acceleration are rad/s²
- When a rigid object rotates about a fixed axis, every portion of the object has the same angular velocity and the same angular acceleration

Analogies Between Linear and Rotational Motion

Rotational Motion About a Fixed Axis with Constant Acceleration	Linear Motion with Constant Acceleration
$\omega = \omega_i + \alpha t$	$v = v_i + at$
$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$	$\Delta x = v_i t + \frac{1}{2}at^2$
$\omega^2 = \omega_i^2 + 2\alpha\Delta\theta$	$v^2 = v_i^2 + 2a\Delta x$

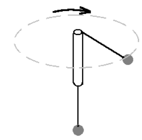
Relationship Between Angular and Linear Quantities

- Displacements
  $s = \theta r$
- Speeds
  $v = \omega r$
- Accelerations
  $a = \alpha r$

- Every point on the rotating object has the same angular motion
- Every point on the rotating object does *not* have the same linear motion

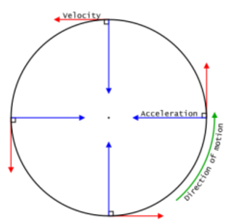
Centripetal Acceleration

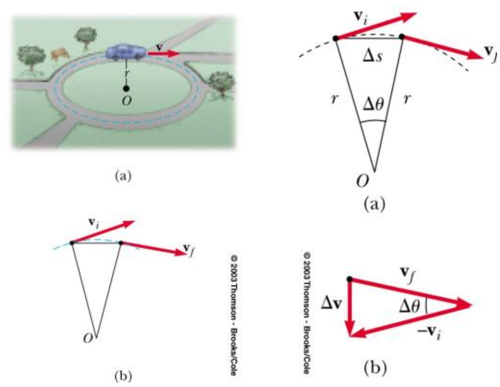
- An object traveling in a circle, even though it moves with a constant speed, will have an acceleration
- The centripetal acceleration is due to the change in the *direction* of the velocity



Centripetal Acceleration, cont.

- Centripetal refers to "center-seeking"
- The direction of the velocity changes
- The acceleration is directed toward the center of the circle of motion

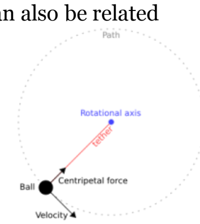




Centripetal Acceleration and Angular Velocity

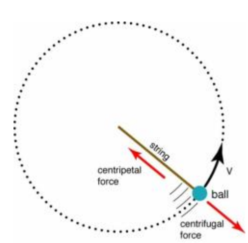
- The angular velocity and the linear velocity are related ($v = \omega r$)
- The centripetal acceleration can also be related to the angular velocity

$a_C = \omega^2 r$



NO Centrifugal Force

- This is a fake force. Made up to explain inertia.



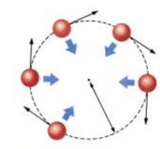
Total Acceleration

- The tangential component of the acceleration is due to changing speed
- The centripetal component of the acceleration is due to changing direction
- Total acceleration can be found from these components

$$a = \sqrt{a_t^2 + a_C^2}$$

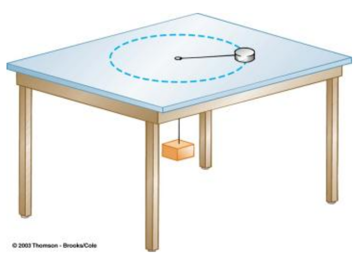
Forces Causing Centripetal Acceleration

- Newton's Second Law says that the centripetal acceleration is accompanied by a force
  - $F = ma_C$
  - F stands for any force that keeps an object following a circular path
    - Tension in a string
    - Gravity
    - Force of friction



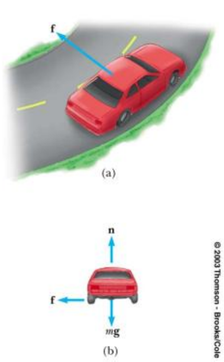
Centripetal Force Example 1

- **What is the mass of the orange block if the cylinder moves in a circle of 30 cm at 6 m/s and the it has a mass of 250g.**



Applications of Forces Causing Centripetal Acceleration

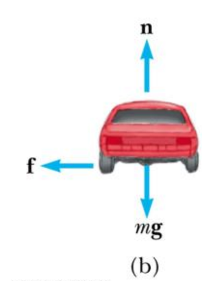
- Many specific situations will use forces that cause centripetal acceleration
  - Level curves
  - Banked curves
  - Horizontal circles
  - Vertical circles



Level Curves

- Friction is the force that produces the centripetal acceleration
- Can find the frictional force, μ, v

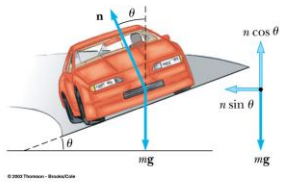
$$v = \sqrt{\mu r g}$$



Banked Curves

- A component of the normal force adds to the frictional force to allow higher speeds

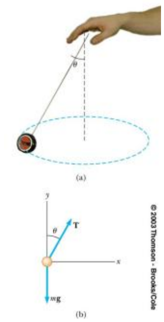
$$\tan\theta = \frac{v^2}{rg}$$



Horizontal Circle

- The horizontal component of the tension causes the centripetal acceleration

$$a_c = g\tan\theta$$

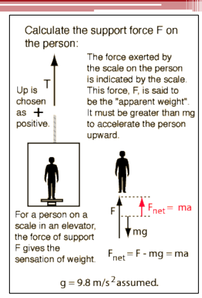


REFESHER: Elevators (From Chapter 4)

In an elevator your weight changes based on the accelerations you are experience.

$T=F_n$=Apparent Weight (+)
F_w=True Weight (-)

$F_n+F_w = F_{net}$ as always $F_{net} / m = a$



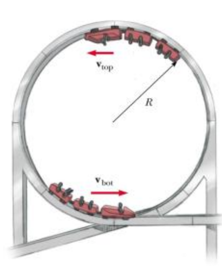
Motion in a Vertical Circle

- We can use Newton's 2nd Law to determine the minimum velocity at the top.
- At the top, the weight must *equal* the centripetal force at the *minimum velocity*. This minimum velocity is also called the *critical velocity* since at any velocity slower, weight will overcome centripetal force and make the object fall out of the circle.

$$mg = \frac{mv_{min}^2}{r}$$ **So then:** $$v_{min} = \sqrt{rg}$$

Vertical Circle

- What is the force at the bottom of the circle on person?



Forces in Accelerating Reference Frames

- Distinguish real forces from fictitious forces
- Centrifugal force is a fictitious force
- Real forces always represent interactions between objects

TABLE 7.3 Useful Planetary Data

Body	Mass (kg)	Mean Radius (m)	Period (s)	Mean Distance from Sun (m)	$\frac{T^2}{r^3}$ (10^{-19} s^2/m^3)
Mercury	3.18×10^{23}	2.43×10^{6}	7.60×10^{6}	5.79×10^{10}	2.97
Venus	4.88×10^{24}	6.06×10^{6}	1.94×10^{7}	1.08×10^{11}	2.99
Earth	5.98×10^{24}	6.38×10^{6}	3.156×10^{7}	1.496×10^{11}	2.97
Mars	6.42×10^{23}	3.37×10^{6}	5.94×10^{7}	2.28×10^{11}	2.98
Jupiter	1.90×10^{27}	6.99×10^{7}	3.74×10^{8}	7.78×10^{11}	2.97
Saturn	5.68×10^{26}	5.85×10^{7}	9.35×10^{8}	1.43×10^{12}	2.99
Uranus	8.68×10^{25}	2.33×10^{7}	2.64×10^{9}	2.87×10^{12}	2.95
Neptune	1.03×10^{26}	2.21×10^{7}	5.22×10^{9}	4.50×10^{12}	2.99
Pluto	$\approx 1 \times 10^{23}$	$\approx 3 \times 10^{6}$	7.82×10^{9}	5.91×10^{12}	2.96
Moon	7.36×10^{22}	1.74×10^{6}	—	—	—
Sun	1.991×10^{30}	6.96×10^{8}	—	—	—

Fig. T7.3, p. 210
Slide 37

Kepler's Laws

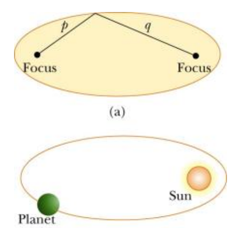
- All planets move in elliptical orbits with the Sun at one of the focal points.
- A line drawn from the Sun to any planet sweeps out equal areas in equal time intervals.
- The square of the orbital period of any planet is proportional to cube of the average distance from the Sun to the planet.

Kepler's Laws, cont.

- Based on observations made by Brahe
- Newton later demonstrated that these laws were consequences of the gravitational force between any two objects together with Newton's laws of motion

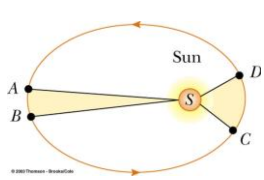
Kepler's First Law

- All planets move in elliptical orbits with the Sun at one focus.
  - Any object bound to another by an inverse square law will move in an elliptical path
  - Second focus is empty



Kepler's Second Law

- A line drawn from the Sun to any planet will sweep out equal areas in equal times
  - Area from A to B and C to D are the same



Kepler's Third Law

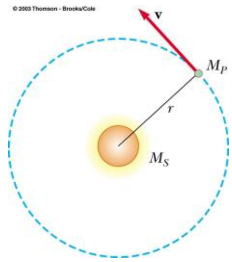
- The square of the orbital period of any planet is proportional to cube of the average distance from the Sun to the planet.

$$T^2 = Kr^3$$

  - For orbit around the Sun, $K_S = 2.97 \times 10^{-19}\ s^2/m^3$
  - K is independent of the mass of the planet

Kepler's Third Law application

- Mass of the Sun or other celestial body that has something orbiting it
- Assuming a circular orbit is a good approximation



Newton's Law of Universal Gravitation

- Every particle in the Universe attracts every other particle with a force that is directly proportional to the product of the masses and inversely proportional to the square of the distance between them.

$$F = G\frac{m_1 m_2}{r^2}$$

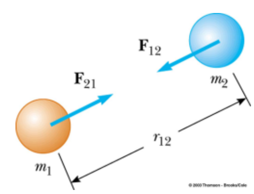
Gravitational Force

- Mutual force of attraction between any two objects
- Expressed by Newton's Law of Universal Gravitation:

$$F_g = G\frac{m_1 m_2}{r^2}$$

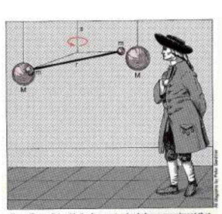
Law of Gravitation, cont.

- G is the constant of universal gravitational
- G = 6.673 x 10^{-11} N m^2 /kg^2
- This is an example of an *inverse square law*



Gravitation Constant

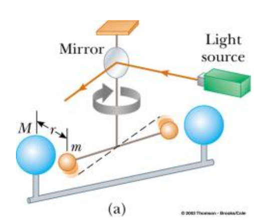
- **Newton could only approximate *G* in his day through astronomical observations. The first precise measurement was made 71 years after Newton died, in 1798 by English scientist Henry Cavendish.**



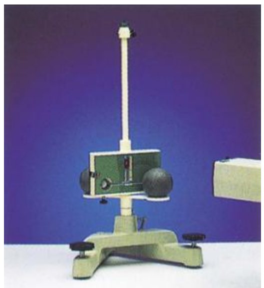
Henry Cavendish with the famous torsion balance experiment that determined the gravitational constant G and demonstrated Newton's inverse-square law of gravitation. Large lead spheres placed close to small ones caused angular deflections

Gravitation Constant

- Determined experimentally
- Henry Cavendish
  - 1798
- The light beam and mirror serve to amplify the motion



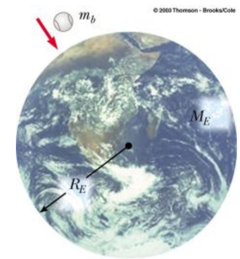
Gravitational Constant Apparatus



Applications of Universal Gravitation

- *Mass of the earth*
- Use an example of an object close to the surface of the earth
  - $r \sim R_E$

$$M_E = \frac{gR_E^2}{G}$$



Applications of Universal Gravitation

- Acceleration due to gravity
- g will vary with altitude

$$g = G\frac{M_E}{r^2}$$

TABLE 7.1

Free-Fall Acceleration *g* at Various Altitudes

Altitude (km)[a]	*g* (m/s²)
1 000	7.33
2 000	5.68
3 000	4.53
4 000	3.70
5 000	3.08
6 000	2.60
7 000	2.23
8 000	1.93
9 000	1.69
10 000	1.49
50 000	0.13

[a] All values are distances above Earth's surface.

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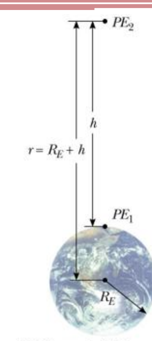
Gravitational Potential Energy

- PE = mgy is valid only near the earth's surface
- For objects high above the earth's surface, an alternate expression is needed

$$PE = -G\frac{M_E m}{r}$$

  - Zero reference level is infinitely far from the earth



Escape Speed

- The escape speed is the speed needed for an object to soar off into space and not return

$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}}$$

- For the earth, v_{esc} is about 11.2 km/s
- Note, v is independent of the mass of the object

TABLE 7.2

Escape Speeds for the Planets and the Moon

Planet	v_{esc} (km/s)
Mercury	4.3
Venus	10.3
Earth	11.2
Moon	2.3
Mars	5.0
Jupiter	60.0
Saturn	36.0
Uranus	22.0
Neptune	24.0
Pluto	1.1

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2 Masses

- Inertial Mass

$$m_{inertial} = \frac{F_{net}}{a}$$

- Gravitational Mass

$$m_{gravitational} = \frac{r^2 F_{grav}}{Gm}$$

Special Relativity (1905)

- Special Relativity is probably the most misunderstood theory of physics. Fewer people know anything about General Relativity or Quantum Mechanics, but for no other theory in physics have so many people been told so many things about the theory that are philosophically important and wrong. It is widely agreed that Einstein taught us that "all things are relative" and "nothing is absolute." Neither of these are any part of Special Relativity, which simply gives transformation rules that allow one person to translate his measurements so that they will *agree* with the measurements of other, moving, people.

Special Relativity (1905)

- Special Relativity does have some strange aspects, all well tested experimentally.
  - nothing moving at a speed less than the speed of light can ever move faster than light.
  - Moving objects contract, *really* contract, not just appear to contract.
  - Time is dialates the closer you are to the speed of light. To the person moving near the speed of light, time is moving at the same rate, but to everyone else, time is moving slower
- Although Einstein denied using the experiment in constructing the theory, logically these phenomena all follow from a difficult experiment done in a scientifically obscure country a decade before the end of the 19th century....

Relativity

- In 1887, Albert Michelson and Edward Morely, working at the Case School of Applied Science in Cleveland, Ohio, performed a clever little experiment that showed that the speed of light measured by any observer is independent of the speed of the source of the light and also independent of the speed of the observer.

Relativity

- The speed of light does *not* depend on the speed of source or observer, and there is no medium "carrying" the light. The Michelson-Morley experiment was the first experiment done in the US that was of any significance to physics since Benjamin Franklin somehow managed to avoid getting himself killed while investigating lightning.

Special Relativity

- The Final result of Special relativity is

$$E = mc^2 + p^2c^2$$

- This equation is only needed for objects moving very fast
- The second part of the equation is only needed for object that are very large

General Relativity (1915)

- In 1907, two years after proposing the special theory of relativity, Einstein was preparing a review of special relativity when he suddenly wondered how Newtonian gravitation would have to be modified to fit in with special relativity. At this point there occurred to Einstein, described by him as the *happiest thought of my life* , namely that an observer who is falling from the roof of a house experiences no gravitational field. He proposed the *Equivalence Principle* as a consequence:-
  - *... we shall therefore assume the complete physical equivalence of a gravitational field and the corresponding acceleration of the reference frame. This assumption extends the principle of relativity to the case of uniformly accelerated motion of the reference frame.*

General Relativity (1915)

- After the major step of the equivalence principle in 1907, Einstein published nothing further on gravitation until 1911. Then he realized that the bending of light in a gravitational field, which he knew in 1907 was a consequence of the equivalence principle, could be checked with astronomical observations. He had only thought in 1907 in terms of terrestrial observations where there seemed little chance of experimental verification. Also discussed at this time is the gravitational red shift, light leaving a massive body will be shifted towards the red by the energy loss of escaping the gravitational field.

General Relativity (1915)

- Einstein published further papers on gravitation in 1912. In these he realized that the Lorentz transformations will not apply in this more general setting. Einstein also realized that the gravitational field equations were bound to be non-linear and the equivalence principle appeared to only hold locally.

General Relativity (1915)

- When Planck visited Einstein in 1913 and Einstein told him the present state of his theories Planck said
  - *"As an older friend I must advise you against it for in the first place you will not succeed, and even if you succeed no one will believe you"*

General Relativity (1915)

- Then on the 18th November, 1915 he made a discovery about which he wrote
  - *For a few days I was beside myself with joyous excitement .*

General Relativity (1915)

- His theory showed that space and time curve.

- The large ball will cause a deformation in the sheet's surface. A baseball dropped onto the sheet will roll toward the bowling ball. Einstein theorized that smaller masses travel toward larger masses not because they are "attracted" by a mysterious force, but because the smaller objects travel through space that is warped by the larger object.

General Relativity (1915)

- **Gravitational Time Dilation**
  - Einstein's Special Theory of Relativity predicted that time does not flow at a fixed rate: moving clocks appear to tick more slowly relative to their stationary counterparts. But this effect only becomes really significant at very high velocities that app roach the speed of light. When "generalized" to include gravitation, the equations of relativity predict that gravity, or the curvature of spacetime by matter, not only stretches or shrinks distances (depending on their direction with respect to the gravitational field) but also w ill appear to slow down or "dilate" the flow of time.

General Relativity (1915)

- In most circumstances in the universe, such **time dilation** is miniscule, but it can become very significant when space-time is curved by a massive object such as a black hole. For example, an observer far from a black hole would observe time passing extremely slowly for an astronaut falling through the hole's boundary. In fact, the distant observer would never see the hapless victim actually fall in. His or her time, as measured by the observer, would appear to stand still. The slowing of time near a very simple black hole has been simulated on supercomputers at NCSA and visualized in a computer-generated animation.

Relativity Conclusion

- Special (1905)
  - Length Contracts
  - Time Dilates
  - The speed of light is the speed limit
  - E=mc^2
- General (1915)
  - Equivalence of masses
  - Gravitational Time Dilation