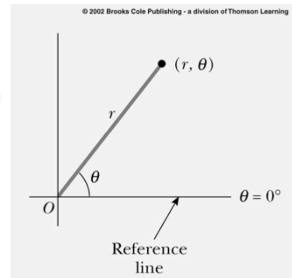
Chapter 3A

Vectors

Polar Coordinates

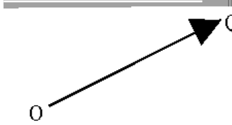
- origin and reference line are noted
- point is distance r from the origin in the direction of angle θ, ccw from reference line
- points are labeled (r,θ)



What is a vector

- A vector is a graphical representation of a mathematical concept
- Every vector has 2 specific quantities
  - Magnitude
    - length
  - Direction
    - angle

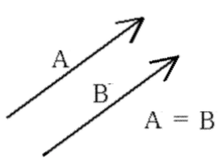
How do we draw it?



- Graphically, a vector is represented by an arrow, defining the direction, and the length of the arrow defines the vector's magnitude. This is shown above. If we denote one end of the arrow by the origin O and the tip of the arrow by Q. Then the vector may be represented algebraically by OQ.

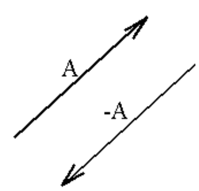
Equal Vectors

- Two vectors, A and **B** are equal if they have the same magnitude and direction, regardless of whether they have the same initial points, as shown in



Opposite Vectors

- A vector having the same magnitude as **A** but in the opposite direction to **A** is denoted by **-A** , as shown to the right



Properties of Vectors

- Equality of Two Vectors
  - Two vectors are **equal** if they have the same magnitude and the same direction
- Movement of vectors in a diagram
  - Any vector can be moved parallel to itself without being affected
- Resultant Vector
  - The **resultant** vector is the sum of a given set of vectors
- Equilibrium Vectors
  - Two vectors are in **Equilibrium** if they have the same magnitude but are 180° apart (opposite directions)
    - **A** = -**B**

Adding Vectors

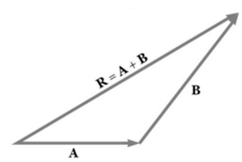
- When adding vectors, their directions must be taken into account
- Units must be the same
- Graphical Methods
  - Use scale drawings
- Algebraic Methods
  - More convenient

Adding Vectors Graphically (Triangle or Polygon Method)

- Choose a scale
- Draw the first vector with the appropriate length and in the direction specified, with respect to a coordinate system
- Draw the next vector with the appropriate length and in the direction specified, with respect to a coordinate system whose origin is the end of vector **A** and parallel to the coordinate system used for **A**

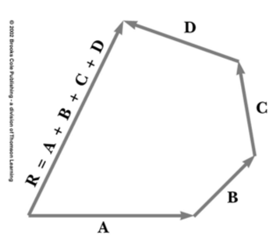
Graphically Adding Vectors, cont.

- Continue drawing the vectors "tip-to-tail"
- The resultant is drawn from the origin of **A** to the end of the last vector
- Measure the length of **R** and its angle
  - Use the scale factor to convert length to actual magnitude



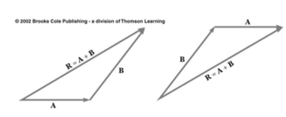
Graphically Adding Vectors, cont.

- When you have many vectors, just keep repeating the process until all are included
- The resultant is still drawn from the origin of the first vector to the end of the last vector



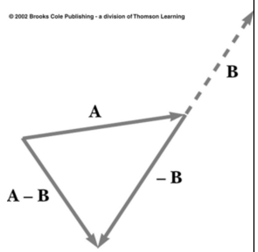
Notes about Vector Addition

- Vectors obey the **Commutative Law of Addition**
  - The order in which the vectors are added doesn't affect the result



Vector Subtraction

- Special case of vector addition
- If **A** – **B**, then use **A** +(**-B**)
- Continue with standard vector addition procedure

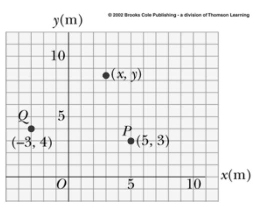


Multiplying or Dividing a Vector by a Scalar

- The result of the multiplication or division is a vector
- The magnitude of the vector is multiplied or divided by the scalar
- If the scalar is positive, the direction of the result is the same as of the original vector
- If the scalar is negative, the direction of the result is opposite that of the original vector

Cartesian coordinate system

- also called rectangular coordinate system
- x- and y- axes
- points are labeled (x,y)



Terrestrial (Cardinal) Coordinates

- The terrestrial coordinate system deals with directions on land.
  - 0 degrees is considered North
  - 90 degrees is considered East
  - 180 degrees is considered South
  - 270 degrees is considered West

The concept of a radian

- A radian is an arc in a circle, equal in length to the radius

- If you start with a circle, then take a line that has the same length as the radius of that circle and bend it around the circle's circumference, it will encompass an angle of one radian.

Radian vs degree

- There are 360 degrees in a circle
- There are 2*Pi radians in a circle

- Therefore 1 radian = 360 degree/(2*Pi)
  - This value is equal to 57.3 degrees

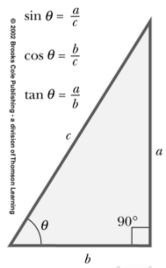
- The conversion factor is then 1 rad=57.3 degrees

SOH CAH TOA

$$\sin(\theta) = \frac{Opp}{Hyp}$$

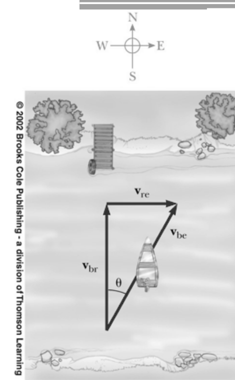
$$\cos(\theta) = \frac{Adj}{Hyp}$$

$$\tan(\theta) = \frac{Opp}{Adj}$$



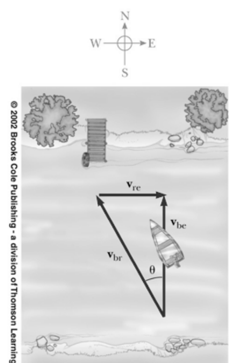
Solve

- Vbr= 10m/s
- Vrc=3m/s
- What is Vbc?



Solve

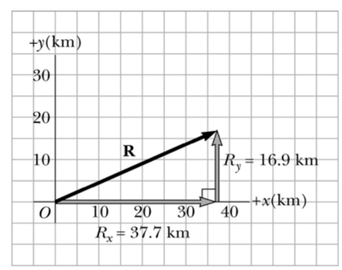
- Vbr= 10m/s
- Vrc=3m/s
- What is theta have to be for the boat to go straight across?



More Trigonometry

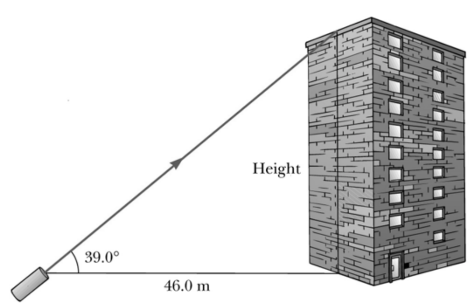
- Pythagorean Theorem

$$c^2 = a^2 + b^2$$



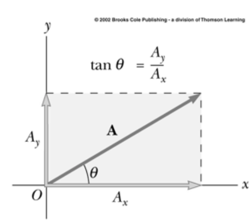
(b)

What is the height



Components of a Vector

- A **component** is a part
- It is useful to use **rectangular components**
  - These are the projections of the vector along the x- and y-axes



Components of a Vector, cont.

- The x-component of a vector is the projection along the x-axis

$$A_x = A\cos\theta$$

- The y-component of a vector is the projection along the y-axis

$$A_y = A\sin\theta$$

- Then,

$$\mathbf{A} = A_x + A_y$$

More About Components of a Vector

- The previous equations are valid ***only if θ is measured with respect to the x-axis***
- The components can be positive or negative and will have the same units as the original vector
- The components are the legs of the right triangle whose hypotenuse is **A**

$$A = \sqrt{A_x^2 + A_y^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{A_y}{A_x}$$

  - May still have to find θ with respect to the positive x-axis

Vector Notation

- Vector notation allows us to treat the components separate in an equation. Just like you wouldn't add together 3x+4y because they are different variables.
- The coordinates (a,b,c) it can be expressed as the sum of three vectors $a\hat{i} + b\hat{j} + c$

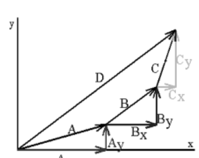
Adding Vectors Example

$$\vec{A} = 3\hat{i} - 5\hat{j} \qquad \vec{B} = -2\hat{i} + 7\hat{j}$$

Adding Vectors Algebraically

- Convert to polar coordinates if not already in Cartesian coordinates and sketch the vectors
- Find the x- and y-components of all the vectors
- Add all the x-components
  - This gives R_x:

$$R_x = \sum v_x$$

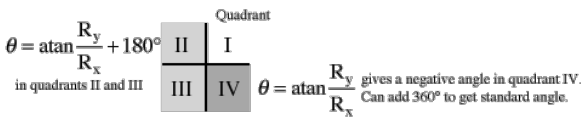


Adding Vectors Algebraically, cont.

- Add all the y-components
  - This gives R_y: $R_y = \sum v_y$
- Use the Pythagorean Theorem to find the magnitude of the Resultant: $R = \sqrt{R_x^2 + R_y^2}$
- Use the inverse tangent function to find the direction of R:

$$\theta = \tan^{-1}\frac{R_y}{R_x}$$

Adding Vectors Algebraically, **HARDEST PART**



Adding Vectors Example #2

$$\vec{R}_1 = 4\hat{i} - 6\hat{j} \qquad \vec{R}_2 = [11, 160^\circ]$$

Terrestrial (Cardinal) Coordinates

- The terrestrial coordinate system deals with directions on land.
  - 0 degrees C.C. is 90 degrees P.C.
  - 90 degrees C.C. is 0 degrees P.C.
  - 180 degrees C.C. is 270 degrees P.C.
  - 270 degrees C.C. is 180 degrees P.C.
- It is best to draw a picture

Terrestrial (Cardinal) Coordinates

- These coordinates are broken into sub coordinates
  - 20 degrees North of East = 20 degrees p.c.
  - 20 degrees East of North = 70 degrees p.c.
    - Note that you start in the second direction and go in the direction of the first.
  - So if you are going west of north, you start in the north and go west

2D Kinematics

Trigonometric Identities

- 2*sin (θ)*cos (θ) = sin (2θ)
- $\sin^2(\theta) + \cos^2(\theta) = 1$
- $\tan^2(\theta) + 1 = \sec^2(\theta)$

Review

- The position of an object is described by its position vector, **r**
- The **displacement** of the object is defined as the ***change in its position***
  - $\Delta \mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$
- The average velocity is the ratio of the displacement to the time interval for the displacement
- The instantaneous velocity is the limit of the average velocity as Δt approaches zero
- The average acceleration is defined as the rate at which the velocity changes
- The instantaneous acceleration is the limit of the average acceleration as Δt approaches zero

Ways an Object Might Accelerate

- The **magnitude** of the velocity (the speed) can change
- The **direction** of the velocity can change
  - Even though the magnitude is constant
- Both the magnitude and the direction can change

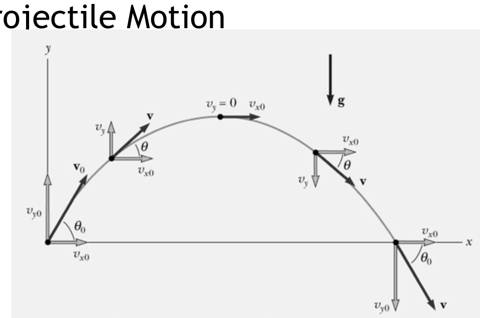
Projectile Motion

- An object may move in both the x and y directions simultaneously
  - It moves in two dimensions
- The form of two dimensional motion we will deal with is called **projectile motion**

Rules of Projectile Motion

- The x- and y-directions of motion can be treated independently
- The x-direction is uniform motion
  - $a_x = 0$
- The y-direction is free fall
  - $a_y = -g$
- The initial velocity can be broken down into its x- and y-components

Projectile Motion



Some Details About the Rules

- x-direction
  - $a_x = 0$
  - $v_{xo} = v_o \cos\theta_o = v_x = \text{constant}$
  - $x = v_{xo}t$
    - This is the only operative equation in the x-direction since there is uniform velocity (NO Acceleration) in that direction

More Details About the Rules

- y-direction
  - $v_{yo} = v_o \sin \theta_o$
  - free fall problem
    - a = -g
  - take the positive direction as upward
  - uniformly accelerated motion, so the motion equations all hold

Time to Vertex

- The apex or Vertex is the top of the trajectory
- Assume up is positive and down is negative
- If we are looking for the time to the vertex we use the following concepts:
  - Kinematics Equation:

$$v_{yf} = v_{yo} + at$$

  - The y component of the projectile motion

$$v_{yo} = v_o \sin \theta_o$$

  - The equations together solved for the velocity at the top of the trajectory:

$$0 = v_{yo} \sin \theta_o + at$$

  - Final time Equation

Time to Vertex

- Simplify

$$-v_o \sin \theta_o = -gt$$

- Final time Equation

$$\frac{v_o \sin \theta_o}{g} = t$$

Total Time of flight (Back to elevation started from)

- Final time Equation

$$\frac{2 * v_o \sin \theta_o}{g} = t$$

Distance to Vertex

- We start with the kinematic equation:

$$vf^2 = vo^2 + 2ad$$

- We know the acceleration is '-g'
- We know the final velocity is 0 m/s
- We know the initial Velocity

$$v_{yo} = v_o \sin \theta_o$$

Distance to Vertex

- Putting these equations together we get

$$0^2 = (v_o \sin \theta_o)^2 + 2(-g)d$$

- Simplifying this down we get

$$\Delta d_y = \frac{(v_o \sin \theta_o)^2}{2g}$$

Distance to Vertex (alternative)

- We start with the kinematic equation:

$$dy = vo*t + \frac{1}{2}at^2$$

- We know t to the vertex:

$$\frac{v_o \sin\theta_o}{g} = t$$

- We know the initial Velocity

$$v_{yo} = v_o \sin\theta_o$$

Distance to Vertex (alternative)

- So we plug them in:

$$dy = (v_o \sin\theta)*(\frac{v_o \sin\theta_o}{g}) + \frac{1}{2}(-g)(\frac{v_o \sin\theta_o}{g})^2$$

- Simplifying it

$$dy = \frac{(v_o \sin\theta_o)^2}{g} - \frac{1}{2}\frac{(v_o \sin\theta_o)^2}{g}$$

- Give us

$$dy = \frac{(v_o \sin\theta_o)^2}{2g}$$!!!!!!

The Range Equation

- We start with the kinematic equation:

$$d_x = vo*t + \frac{1}{2}at^2$$

- We know t total is:

$$\frac{2v_o \sin\theta_o}{g} = t$$

- We know the initial Velocity

$$v_{xo} = v_o \cos\theta_o$$

- We know a_x=0

The Range Equation

- Putting these equations together we get

$$d_x = v_o \cos\theta * \frac{v_o \sin\theta_o}{g} + \frac{1}{2}*0*(\frac{v_o \sin\theta_o}{g})^2$$

- Simplifying this down we get

$$d_x = \frac{v_o^2 \sin\theta_o \cos\theta_0}{g}$$

- From a trigonometric Identity we get

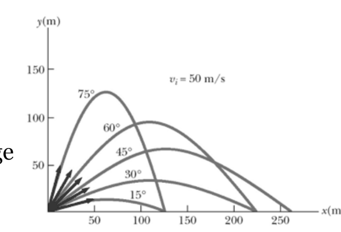
$$\Delta x = \frac{v_o^2 \sin(2\theta_o)}{g} = Range\_Equation$$

Physics Applets

- http://physics.uwstout.edu/physapplets/javapm/java/bounce1/index.html
- http://planck.phys.uwosh.edu/rioux/physlets/kinematics_7.html
- http://www.ngsir.netfirms.com/englishhtm/ThrowABall.htm
- http://physics.usask.ca/~pywell/p121/Notes/projectile/projectile.html
- http://jersey.uoregon.edu/vlab/Cannon/index.html
- http://www.walter-fendt.de/ph11e/projectile.htm

Projectile Motion at Various Initial Angles

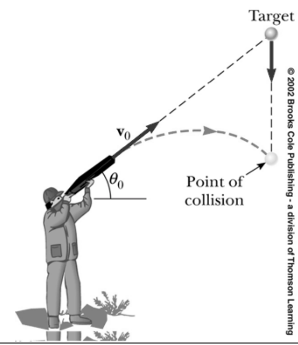
- Complementary values of the initial angle result in the same range
  - The heights will be different
- The maximum range occurs at a projection angle of 45°



Other Equations

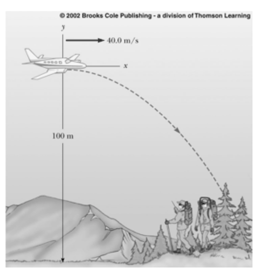
- These equations only work when an object is shot and lands at the same height. What happens when it lands at a different height, well the equations get harder. We need to use out concepts from chapter 5 to work these type of equations out. Here are some examples.

Interesting Fact



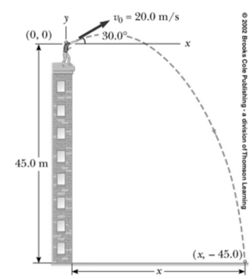
Some Variations of Projectile Motion

- An object may be fired horizontally
- The initial velocity is all in the x-direction
  - $v_o = v_x$ and $v_y = 0$
- All the general rules of projectile motion apply

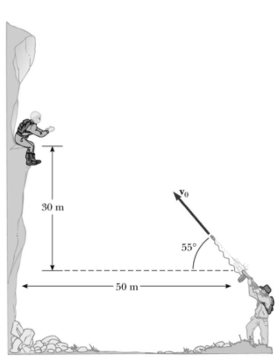


Non-Symmetrical Projectile Motion

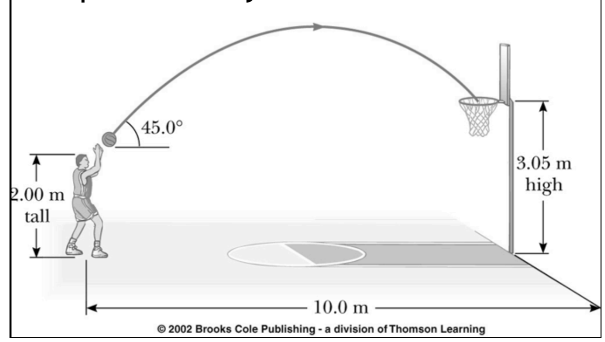
- Follow the general rules for projectile motion
- Break the y-direction into parts
  - up and down
  - symmetrical back to initial height and then the rest of the height



Explain what you could do to solve this problem



Explain what you could do to solve



Velocity of the Projectile

- The velocity of the projectile at any point of its motion is the vector sum of its x and y components at that point

$$v = \sqrt{v_x^2 + v_y^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{v_y}{v_x}$$

Relative Velocity

- Relative velocity is about relating the measurements of two different observers
- It is important to specify the frame of reference, since the motion may be different in different frames of reference
- There are no specific equations to learn to solve relative velocity problems

Relative Velocity Notation

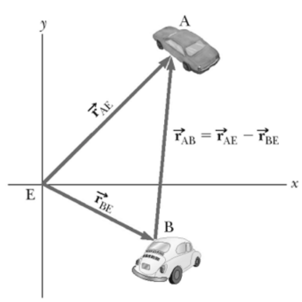
- The pattern of subscripts can be useful in solving relative velocity problems
- Assume the following notation:
  - E is an observer, stationary with respect to the earth
  - A and B are two moving cars

Relative Position Equations

- $\vec{r}_{AE}$ is the position of car A as measured by E
- $\vec{r}_{BE}$ is the position of car B as measured by E
- $\vec{r}_{AB}$ is the position of car A as measured by car B
- $\vec{r}_{AB} = \vec{r}_{AE} - \vec{r}_{EB}$

Relative Position

- The position of car A relative to car B is given by the vector subtraction equation



Relative Velocity Equations

- The rate of change of the displacements gives the relationship for the velocities

$$\vec{v}_{AB} = \vec{v}_{AE} - \vec{v}_{EB}$$