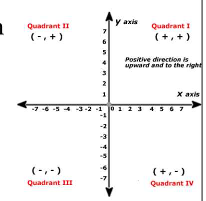
Chapter 2

Motion in One Dimension

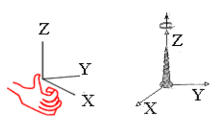
Cartesian Coordinate System

- The most common coordinate system for representing positions in space is one based on three perpendicular spatial axes generally designated x, y, and z.
- Any point P may be represented by three signed numbers, usually written (x, y, z) where the coordinate is the perpendicular distance from the plane formed by the other two axes.



Right Hand Rule

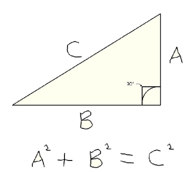
- Point the fingers of your right hand down the positive X axis
- Turn your fingers inward towards the positive Y axis
- Your thumb is now pointing in the positive Z axis

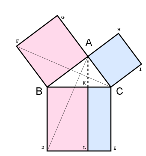


Right Hand Rule

Pythagorean Theorem

- $a^2 + b^2 = c^2$
- $x^2 + y^2 = z^2$





Distance Formula

- 3-D

$$d = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$$

- 2-D

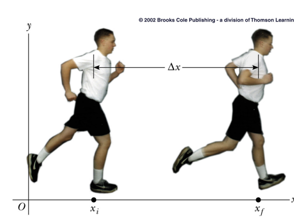
$$d = \sqrt{\Delta x^2 + \Delta y^2}$$

- 1-D

$$d = \sqrt{\Delta x^2} = \Delta x$$

Position

- Defined in terms of a **frame of reference**
  - One dimensional, so generally the x- or y-axis
- Point Particle



Scalar Quantities

- Scalar quantities are completely described by magnitude only
  - Examples:
    - 35 meters
    - 40 miles per hour
    - 20 kilograms

Vector Quantities

- Vector quantities need both magnitude (size) and direction to completely describe them
- Examples
    - 35 miles per hour north
    - 20 meters east
- We can draw vectors
  - Represented by an arrow, the length of the arrow is proportional to the magnitude of the vector
  - Head of the arrow represents the direction
- We can identify vector variables:
  - Generally printed in bold face type

Displacement

- Measures the change in position
  - Represented as Δx (if horizontal) or Δy (if vertical)
  - Vector quantity
    - + or - is generally sufficient to indicate direction for one-dimensional motion

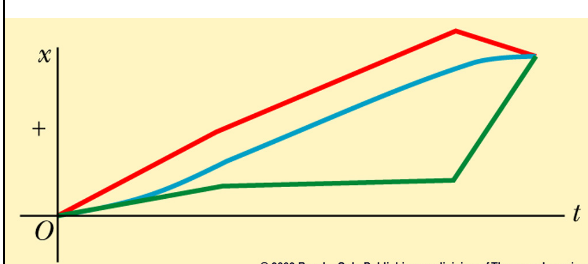
  - Units are meters (m)

Distance

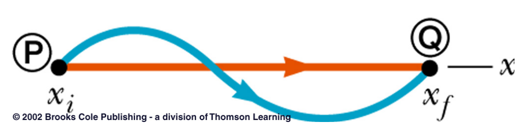
- Measures how far something has traveled
  - Scalar quantity
    - Ignore signs take the absolute values of ever measurement and add them together.

  - Units are meters (m) in SI, centimeters (cm) in cgs or feet (ft) in US Customary

Displacement

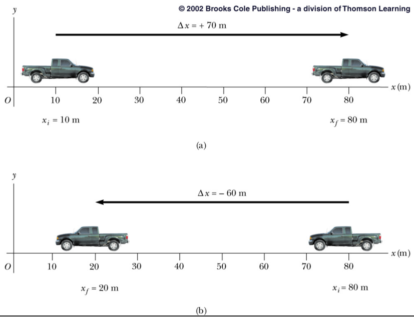


Distance



- Distance may be, but is not necessarily, the magnitude of the displacement
- Blue line shows the distance
- Red line shows the displacement

Displacements vs Distance



The idea of variables

- Do not think that a variable always stands for the same concept.
- You must evaluate what you are looking at.
- Example
  - Displacement is represented by (d, x, s, y, z…)

Velocity

- It takes time for an object to undergo a displacement
- The **average velocity** is rate at which the displacement occurs

$$v_{average} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

- generally use a time interval, so to let $t_i = 0$
- V(bar) = V average

Velocity continued

- Direction will be the same as the direction of the displacement (time interval is always positive)
  - + or - is sufficient
- Units of velocity are m/s
- Other units may be given in a problem, but generally will need to be converted to these

Speed

- The **average speed** of an object is defined as the total distance traveled divided by the total time elapsed

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$

$$s = \frac{d}{t}$$

  - Speed is a scalar quantity

Speed, cont

- Average speed totally ignores any variations in the object's actual motion during the trip
- May be, but is not necessarily, the magnitude of the velocity
- The total distance and the total time are all that is important
- SI units are m/s
  - same units as velocity

Slope

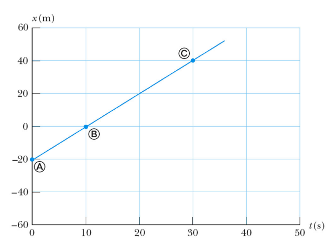
$$Slope = \frac{Rise}{Run} = \frac{\Delta y}{\Delta x} = \frac{y_f - y_i}{x_f - x_i}$$

Graphical Interpretation of Velocity

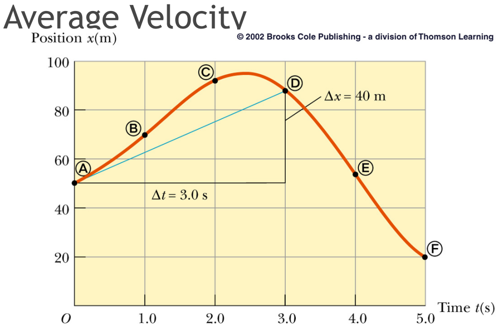
- Velocity can be determined from a position-time graph
- Average velocity equals the slope of the line joining the initial and final positions
- Compare units of slope
- Instantaneous velocity is the slope of the tangent to the curve at the time of interest
- The instantaneous speed is the magnitude of the instantaneous velocity

Average Velocity, Constant

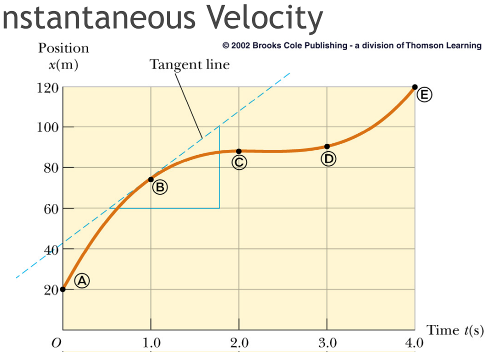
- The straight line indicates constant velocity
- The slope of the line is the value of the average velocity



Average Velocity



Instantaneous Velocity



Instantaneous Velocity

- The limit of the average velocity as the time interval becomes infinitesimally short, or as the time interval approaches zero

$$v \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

- The instantaneous velocity indicates what is happening at every point of time

Instantaneous Velocity on a Graph

- The slope of the line tangent to the position-vs.-time graph is defined to be the instantaneous velocity at that time
  - The instantaneous speed is defined as the magnitude of the instantaneous velocity

Uniform Velocity

- Uniform velocity is constant velocity
- If the instantaneous velocities are always the same
  - All the instantaneous velocities will also equal the average velocity

Acceleration

- Changing velocity (non-uniform) means an acceleration is present
- Acceleration is the rate of change of the velocity

$$a_{average} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t}$$

- Units are m/s^2 (SI), cm/s^2 (cgs), and ft/s^2 (US Cust)

Acceleration

- Negative acceleration is directional
- Deceleration is just acceleration in the negative direction. Decelerations does not exist

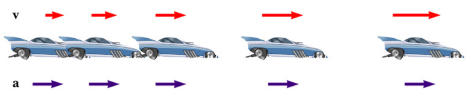
Relationship Between Acceleration and Velocity

- Uniform velocity (shown by red arrows maintaining the same size)
- Acceleration equals zero

Time Intervals and displacement

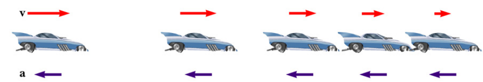
- Objects that are moving equal distances in equal times are moving at constant velocity
- Objects that are not moving equal distances in equal times are accelerating in the positive or negative direction

Relationship Between Velocity and Acceleration



- Velocity and acceleration are in the same direction
- Acceleration is uniform (blue arrows maintain the same length)
- Velocity is increasing (red arrows are getting longer)

Relationship Between Velocity and Acceleration



- Acceleration and velocity are in opposite directions
- Acceleration is uniform (blue arrows maintain the same length)
- Velocity is decreasing (red arrows are getting shorter)

Velocity and Acceleration Equations Revisited

$$v_{average} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

$$a_{average} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t}$$

Average Acceleration

- Vector quantity
- Compare dimensions of velocity vs time graph
- When the sign of the velocity and the acceleration are the same (either positive or negative), then the speed is increasing
- When the sign of the velocity and the acceleration are in the opposite directions, the speed is decreasing

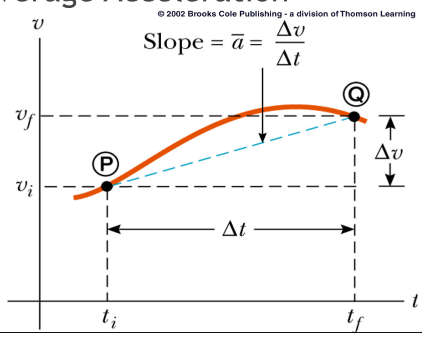
Instantaneous and Uniform Acceleration

- The limit of the average acceleration as the time interval goes to zero
- When the instantaneous accelerations are always the same, the acceleration will be uniform
  - The instantaneous accelerations will all be equal to the average acceleration

Graphical Interpretation of Acceleration

- Average acceleration is the slope of the line connecting the initial and final velocities on a velocity-time graph
- Instantaneous acceleration is the slope of the tangent to the curve of the velocity-time graph

Average Acceleration

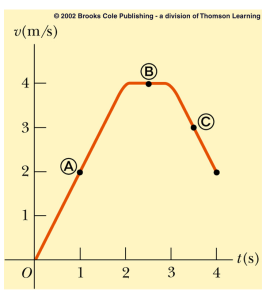


Area Under Curves

- The area under a velocity vs time curve is equivalent to the displacement.
- Using the area of a triangle formula
  - A=1/2 bh

  We can determine the displacement from a velocity time graph.

Area Under Curve Practice



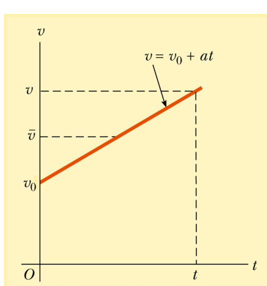
Derivation of Kinematic equation 1

$$a_{average} = \frac{v_f - v_i}{\Delta t}$$

$$\bar{a}\Delta t = v_f - v_i$$

$$\bar{a}\Delta t + v_i = v_f$$

Graphical Derivation of Kinematics 1



Kinematic Equation 1

$$V_f = v_o + a * t$$

Derivation of Kinematics Equation 2

We know Average Velocity

$$\bar{v} = \frac{\Delta d}{\Delta t}$$

Average Velocity is also

$$\bar{v} = \frac{v_o + v_f}{2}$$

But only when acceleration is constant

Derivation of Kinematics Equation 2

Therefore putting it together:

$$\frac{\Delta d}{\Delta t} = \frac{v_0 + v_f}{2}$$

Manipulating this equation

$$\Delta d = \frac{1}{2}(v_0 + v_f)\Delta t$$

Keep this for Kinematic 3

Derivation of Kinematics Equation 2

Now substitute Kinematic 1 in for vf

$$\Delta d = \frac{1}{2}(v_0 + (v_0 + at))\Delta t$$

Now simplify the equation and you get:

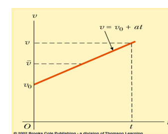
$$\Delta d = v_o * t + \frac{1}{2}at^2$$

Kinematics 2 Alternative Derivation

-If an object is not accelerating then:

$$d = v_o * t$$

Remember that d=area under v vs t graph



$$area\_of\_Triangle = 1/2(b)(h)$$
$$area\_of\_Triangle = 1/2(t)(v_f - v_o)$$

Combining v_0t we get

$$d = v_o * t + area\_of\_Triangle$$
$$d = v_o * t + \frac{1}{2}(v_f - v_o) * t$$

Kinematics 2 Alternative Derivation

$$\bar{at} + v_o = v_f$$

$$d = \frac{1}{2}(vf - vo) * t$$

Combining the Equation Above we get:

$$\Delta d = v_o * t + \frac{1}{2}at^2$$

Derivation of Kinematics Equation 3

We start with: $d = \frac{1}{2}(v_f + v_0)t$

From Kinematic 1 we get

$$t = (v_f - v_i)/a$$

Putting them together we get:

$$d = \frac{1}{2}(v_0 + v_f)(v_0 - v_f)/a$$

Derivation of Kinematics Equation 3

Simplifying the equation we get

$$2ad = v_0^2 - v_f^2$$

Solving for v_f we get:

$$v_f^2 = v_o^2 + 2a\Delta d$$

IMPORTANT

- Make Sure the Table 2.3 (Equation of motion for uniform acceleration) makes it onto your equation sheet

TABLE 2.3 Equations for Motion in a Straight Line Under Constant Acceleration

Equation	Information Given by Equation
$v = v_0 + at$	Velocity as a function of time
$\Delta x = \frac{1}{2}(v_0 + v)t$	Displacement as a function of velocity and time
$\Delta x = v_0 t + \frac{1}{2}at^2$	Displacement as a function of time
$v^2 = v_0^2 + 2a\,\Delta x$	Velocity as a function of displacement

Note: Motion is along the x axis. At $t = 0$, the velocity of the particle is v_0.

Quadratic Equation

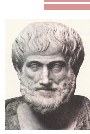
- Sometimes we are going to find that we have a quadratic trinomial equation we can't solve. So we us the quadratic solving equation to solve it.
  - The equation should be put into the form:
    - $ax^2 + bx + c = 0$
  - Then solve it with

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Aristotle

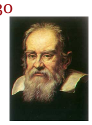
- Broke motion into 2 parts
  - Natural Motion
    - Things always try to return to their proper place
    - Objects falling speed proportionate to their weight
  - Violent Motion
    - Pushing
    - Pulling
- Earth Centered Universe

Time Line

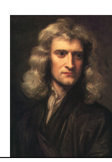


- Aristotle 384-322 BC
- Eratosthenes 276 BC – 200 BC
- Copernicus 2/19/1473- 5/24/1543

- Kepler 12/27/1571- 1630



- Galileo 1564-1642



- Newton 1/4/1643 – 3/31/1727

Free Fall

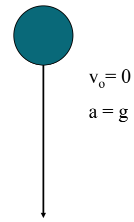
- All objects moving under the influence of only gravity are said to be in free fall
- All objects falling near the earth's surface fall with a constant acceleration
- Galileo originated our present ideas about free fall from his inclined planes
- The acceleration is called the acceleration due to gravity, and indicated by g

Acceleration due to Gravity

- Symbolized by g
- g = 9.8 m/s^2
- g is always directed downward
  - toward the center of the earth

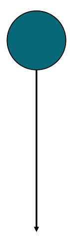
Free Fall -- an object dropped

- Initial velocity is zero
- Let up be positive
- Use the kinematic equations
  - Generally use y instead of x since vertical



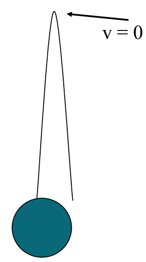
Free Fall -- an object thrown downward

- a = g
- Initial velocity ≠ 0
  - With upward being positive, initial velocity will be negative



Free Fall -- object thrown upward

- Initial velocity is upward, so positive
- The instantaneous velocity at the maximum height is zero
- a = g everywhere in the motion
  - g is always downward, negative

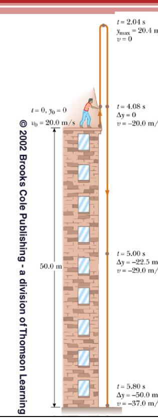


Thrown upward, cont.

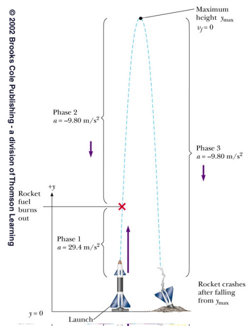
- The motion may be symmetrical
  - then $t_{up} = t_{down}$
  - then $v_f = -v_o$
- The motion may not be symmetrical
  - Break the motion into various parts
    - generally up and down

Non-symmetrical Free Fall

- Need to divide the motion into segments
- Possibilities include
  - Upward and downward portions
  - The symmetrical portion back to the release point and then the non-symmetrical portion

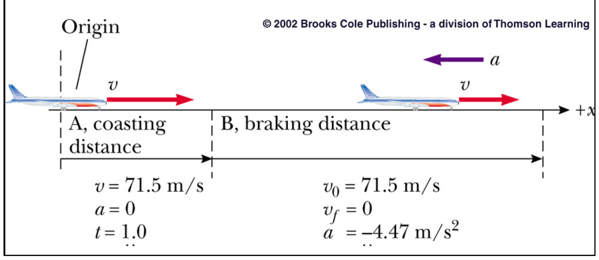


Combination Motions



Additional Problems

How far does the plane go in the first second? How about once it starts breaking?



How long does it take the trooper to catch up?