


16 Relativity—Momentum, Mass, Energy, and Gravity Conceptual Physics

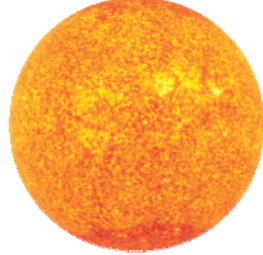


**THE BIG IDEA**

According to special relativity, mass and energy are equivalent. According to general relativity, gravity causes space to become curved and time to undergo changes.

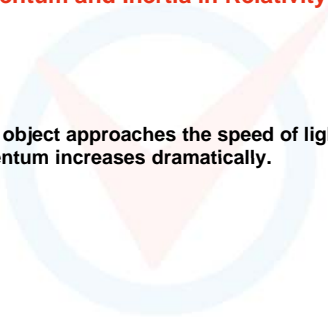
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One of the most celebrated outcomes of special relativity is the discovery that mass and energy are one and the same thing—as described by  $E = mc^2$ . Einstein's *general theory of relativity*, developed a decade after his special theory of relativity, offers another celebrated outcome, an alternative to Newton's theory of gravity.



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### 16.1 Momentum and Inertia in Relativity




As an object approaches the speed of light, its momentum increases dramatically.

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### 16.1 Momentum and Inertia in Relativity

If we push an object that is free to move, it will accelerate.  
 If we push with a greater and greater force, we expect the acceleration in turn to increase.  
 It might seem that the speed should increase without limit, but there is a speed limit in the universe—the speed of light.

At least one thing reaches the speed of light—light itself! But a particle of light has no rest mass. A material particle can never be brought to the speed of light. Light can never be brought to rest.



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### 16.1 Momentum and Inertia in Relativity

#### Newtonian and Relativistic Momentum

Recall Newton's second law, expressed in terms of momentum:

$$F = \Delta mv / \Delta t$$

(which reduces to the familiar  $F = ma$ , or  $a = F/m$ ).  
 Apply more impulse and the object acquires more momentum.  
 Momentum can increase without any limit, even though speed cannot.

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### 16.1 Momentum and Inertia in Relativity

Momentum equals mass times velocity:

$$p = mv$$

(we use  $p$  for momentum)  
 To Newton, infinite momentum would mean infinite speed.  
 Einstein showed that a new definition of momentum is required:

$$p = \frac{mv}{\sqrt{1 - (v^2/c^2)}}$$

where  $v$  is the speed of an object and  $c$  is the speed of light.

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### 16.1 Momentum and Inertia in Relativity

This is **relativistic momentum**, which is noticeable at speeds approaching the speed of light.

The relativistic momentum of an object of mass  $m$  and speed  $v$  is larger than  $mv$  by a factor of  $\frac{1}{\sqrt{1-(v^2/c^2)}}$ .

$$p = \frac{mv}{\sqrt{1-(v^2/c^2)}}$$

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### 16.1 Momentum and Inertia in Relativity

As  $v$  approaches  $c$ , the denominator approaches zero. This means that the momentum approaches infinity!

An object pushed to the speed of light would have infinite momentum and would require an infinite impulse, which is clearly impossible.

$$p = \frac{mv}{\sqrt{1-(v^2/c^2)}}$$

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### 16.1 Momentum and Inertia in Relativity

So nothing that has mass can be pushed to the speed of light. Hence  $c$  is the speed limit in the universe.

If  $v$  is much less than  $c$ , the denominator of the equation is nearly equal to 1 and  $p$  is nearly equal to  $mv$ .

Newton's definition of momentum is valid at low speed.

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### 16.1 Momentum and Inertia in Relativity

#### Trajectory of High-Speed Particles

When a particle is pushed close to the speed of light, it acts as if its mass were increasing, because its momentum increases more than its speed increases.

The **rest mass** of an object,  $m$  in the equation for relativistic momentum, is a constant, a property of the object no matter what speed it has.

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### 16.1 Momentum and Inertia in Relativity

When subatomic particles are pushed to nearly the speed of light, their momenta may be thousands of times more than the Newton expression  $mv$  predicts.

Look at the momentum of a high-speed particle in terms of the "stiffness" of its trajectory.

The more momentum a particle has, the harder it is to deflect it—the "stiffer" is its trajectory. If the particle has a lot of momentum, it more greatly resists changing course.

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### 16.1 Momentum and Inertia in Relativity

When a beam of electrons is directed into a magnetic field, the charged particles experience a force that deflects them from their normal paths.

- For a particle with a small momentum, the path curves sharply.
- For a particle with a large momentum, the path curves only a little—its trajectory is "stiffer."
- A particle moving only a little faster than another (99.9% of  $c$  instead of 99% of  $c$ ) will have much greater momentum and will follow a straighter path in the magnetic field.

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### 16.1 Momentum and Inertia in Relativity

If the momentum of the electrons were equal to the Newtonian value of momentum,  $mv$ , the beam would follow the dashed line. The beam instead follows the “stiffer” trajectory shown by the solid line because the relativistic momentum is greater.

The diagram shows an electron beam passing between two electromagnets. A dashed line represents the path if Newtonian momentum applied, and a solid line shows the actual path where relativistic momentum is greater, causing a larger deflection towards the screen.

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### 16.1 Momentum and Inertia in Relativity

Physicists working with subatomic particles at atomic accelerators verify every day the correctness of the relativistic definition of momentum and the speed limit imposed by nature.

At ordinary speeds, an object's momentum is simply its classical value,  $mv$ . For example, at 30 m/s (0.0000001c), the relativistic momentum differs from the classical value by less than one trillionth of a percent. Newton's definition of momentum is valid at low speeds.

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### 16.1 Momentum and Inertia in Relativity

**CONCEPT CHECK:** How does an object's momentum change as it approaches the speed of light?

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### 16.2 Equivalence of Mass and Energy

Mass and energy are equivalent—anything with mass also has energy.

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### 16.2 Equivalence of Mass and Energy

A remarkable insight of Einstein's special theory of relativity is his conclusion that mass is simply a form of energy.

A piece of matter has an “energy of being” called its **rest energy**.

Einstein concluded that it takes energy to make mass and that energy is released when mass disappears. Rest mass is, in effect, a kind of potential energy.

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### 16.2 Equivalence of Mass and Energy

#### Conversion of Mass to Energy

The amount of rest energy  $E$  is related to the mass  $m$  by the most celebrated equation of the twentieth century:

$$E = mc^2$$

where  $c$  is again the speed of light. This equation gives the total energy content of a piece of stationary matter of mass  $m$ .

$E = mc^2$  says that mass is congealed energy. Mass and energy are two sides of the same coin.

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### 16.2 Equivalence of Mass and Energy

The quantity  $c^2$  is a “conversion factor.”

- It converts the measurement of mass to the measurement of equivalent energy.
- It is the ratio of rest energy to mass:  $E/m = c^2$ .
- It has nothing to do with light and nothing to do with motion.

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### 16.2 Equivalence of Mass and Energy

The speed of light  $c$  is a large quantity and its square is even larger. This means that a small amount of mass stores a large amount of energy.

- The magnitude of  $c^2$  is 90 quadrillion ( $9 \times 10^{16}$ ) joules per kilogram.

One kilogram of matter has an “energy of being” equal to 90 quadrillion joules.

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### 16.2 Equivalence of Mass and Energy

#### Examples of Mass-Energy Conversions


Rest energy can be converted to other forms. For example, when we strike a match, a chemical reaction occurs and heat is released.

- The molecules containing phosphorus in a match head rearrange themselves and combine with oxygen to form new molecules.
- These molecules have very slightly less mass than the separate phosphorus- and oxygen-containing molecules by about one part in a billion.
- For all chemical reactions that give off energy, there is a corresponding decrease in mass.

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### 16.2 Equivalence of Mass and Energy

In one second, 4.5 million tons of rest mass is converted to radiant energy in the sun.



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### 16.2 Equivalence of Mass and Energy

In nuclear reactions, rest mass decreases by about 1 part in 1000.


The sun is so massive that in a million years only one ten-millionth of the sun’s rest mass will have been converted to radiant energy.

The present stage of thermonuclear fusion in the sun has been going on for the past 5 billion years, and there is sufficient hydrogen fuel for fusion to last another 5 billion years.

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### 16.2 Equivalence of Mass and Energy

Saying that a power plant delivers 90 million megajoules of energy to its consumers is equivalent to saying that it delivers 1 gram of energy to its consumers, because mass and energy are equivalent.



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### 16.2 Equivalence of Mass and Energy

$E = mc^2$  is not restricted to chemical and nuclear reactions. A change in energy of any object at rest is accompanied by a change in its mass.

- A light bulb filament has more mass when it is energized with electricity than when it is turned off.
- A hot cup of tea has more mass than the same cup of tea when cold.
- A wound-up spring clock has more mass than the same clock when unwound.

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### 16.2 Equivalence of Mass and Energy

These examples involve incredibly small changes in mass—too small to be measured by conventional methods. The equation  $E = mc^2$  is more than a formula for the conversion of rest mass into other kinds of energy, or vice versa. It states that energy and mass are the *same thing*.

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### 16.2 Equivalence of Mass and Energy

**think!**

Can we look at the equation  $E = mc^2$  in another way and say that matter transforms into pure energy when it is traveling at the speed of light squared?

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### 16.2 Equivalence of Mass and Energy

**think!**

Can we look at the equation  $E = mc^2$  in another way and say that matter transforms into pure energy when it is traveling at the speed of light squared?

*Answer:*

No, no, no! Matter cannot be made to move at the speed of light, let alone the speed of light squared (which is not a speed!). The equation  $E = mc^2$  simply means that energy and mass are “two sides of the same coin.”

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### 16.2 Equivalence of Mass and Energy

**CONCEPT CHECK:** What is the relationship between mass and energy?

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### 16.3 The Correspondence Principle

According to the correspondence principle, if the equations of special relativity (or any other new theory) are to be valid, they must correspond to those of Newtonian mechanics—classical mechanics—when speeds much less than the speed of light are considered.

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### 16.3 The Correspondence Principle

If a new theory is to be valid, it must account for the verified results of the old theory.

The **correspondence principle** states that new theory and old must overlap and agree in the region where the results of the old theory have been fully verified.

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### 16.3 The Correspondence Principle

The relativity equations for time dilation, length contraction, and momentum are

$$t = \frac{t_0}{\sqrt{1 - (v^2/c^2)}}$$

$$L = L_0 \sqrt{1 - (v^2/c^2)}$$

$$p = \frac{mv}{\sqrt{1 - (v^2/c^2)}}$$

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### 16.3 The Correspondence Principle


These equations reduce to a Newtonian value for speeds that are very small compared with  $c$ . Then, the ratio  $(v/c)^2$  is very small, and may be taken to be zero. The relativity equations become

$$t = \frac{t_0}{\sqrt{1 - 0}} = t_0$$

$$L = L_0 \sqrt{1 - 0} = L_0$$

$$p = \frac{mv}{\sqrt{1 - 0}} = mv$$

Equations remind us that you can never change only one thing. Change a term on one side of an equation and you change something on the other side.



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### 16.3 The Correspondence Principle

So for everyday speeds:

- The time scales and length scales of moving objects are essentially unchanged.
- The Newtonian equations for momentum and kinetic energy hold true.

When the speed of light is approached, things change dramatically.


The equations of special relativity hold for all speeds, although they are significant only for speeds near the speed of light.

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### 16.3 The Correspondence Principle

Einstein never claimed that accepted laws of physics were wrong, but instead showed that the laws of physics implied something that hadn't before been appreciated.

Much of nature is built on patterns, and looking for those patterns is the primary preoccupation of both artists and scientists. We connect things that were always there but never put together in our thinking.



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### 16.3 The Correspondence Principle

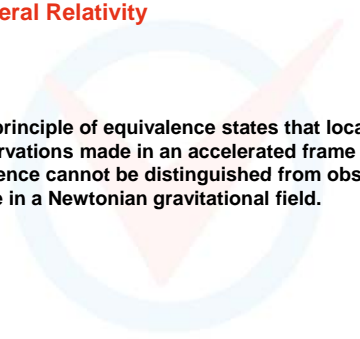
**CONCEPT CHECK:** How does the correspondence principle apply to special relativity?



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### 16.4 General Relativity

 The principle of equivalence states that local observations made in an accelerated frame of reference cannot be distinguished from observations made in a Newtonian gravitational field.



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### 16.4 General Relativity

The special theory of relativity is about motion observed in uniformly moving frames of reference.

Einstein was convinced that the laws of nature should be expressed in the same form in *every* frame of reference.

This motivation led him to develop the **general theory of relativity**—a new theory of gravitation, in which gravity causes space to become curved and time to slow down.

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### 16.4 General Relativity

Einstein was led to this new theory of gravity by thinking about observers in accelerated motion.

He imagined a spaceship far away from gravitational influences.

In such a spaceship at rest or in uniform motion relative to the distant stars, everything within the ship would float freely.

Einstein actually imagined himself in elevators, certainly more common at the time than spaceships.



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### 16.4 General Relativity

If rocket motors were activated to accelerate the ship, things would be different—phenomena similar to gravity would be observed.


The wall adjacent to the rocket motors (the “floor”) would push up against any occupants and give them the sensation of weight.

If the acceleration of the spaceship were equal to  $g$ , the occupants could be convinced the ship was at rest on the surface of Earth.

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### 16.4 General Relativity

a. Everything inside is weightless when the spaceship isn't accelerating.

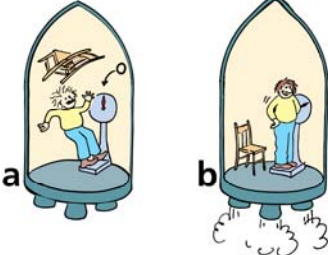


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### 16.4 General Relativity

a. Everything inside is weightless when the spaceship isn't accelerating.

b. When the spaceship accelerates, an occupant inside feels “gravity.”



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## 16.4 General Relativity

### The Principle of Equivalence

Einstein concluded, in what is now called the **principle of equivalence**, that gravity and accelerated motion through space-time are related.

You cannot tell whether you are being pulled by gravity or being accelerated. The effects of gravity and acceleration are equivalent.

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## 16.4 General Relativity

Einstein considered the consequence of dropping two balls, say one of wood and the other of lead, in a spaceship.

- When released, the balls continue to move upward side by side with the velocity that the ship had at the moment of release.
- If the ship were moving at *constant velocity* (zero acceleration), the balls would appear to remain suspended in the same place.

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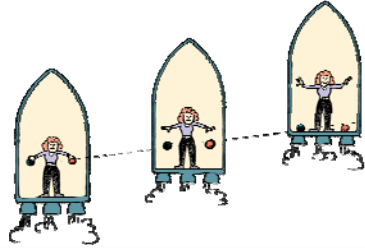
## 16.4 General Relativity

- If the ship were accelerating, the floor would move upward faster than the balls, which would be intercepted by the floor.
- Both balls, regardless of their masses, would meet the floor at the same time.
- Occupants of the spaceship might attribute their observations to the force of gravity.

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## 16.4 General Relativity

To an observer inside the accelerating ship, a lead ball and a wooden ball accelerate downward together when released, just as they would if pulled by gravity.



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## 16.4 General Relativity

Both interpretations of the falling balls are equally valid.

Einstein incorporated this equivalence, or impossibility of distinguishing between gravitation and acceleration, in the foundation of his general theory of relativity.

Einstein stated that the principle holds for all natural phenomena, including optical, electromagnetic, and mechanical phenomena.

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## 16.4 General Relativity

### Bending of Light by Gravity

Consider a ball thrown sideways in a stationary spaceship in the absence of gravity.

The ball will follow a straight-line path relative to both an observer inside the ship and to a stationary observer outside the spaceship.




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### 16.4 General Relativity

If the ship is accelerating, the floor overtakes the ball and it hits the wall below the level at which it was thrown.

- An observer outside the ship still sees a straight-line path.
- An observer in the accelerating ship sees that the path is curved.

The same holds true for a beam of light. The only difference is in the amount of path curvature.

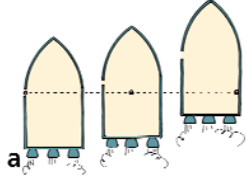



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### 16.4 General Relativity

A ball is thrown sideways in an accelerating spaceship in the absence of gravity.

a. An outside observer sees the ball travel in a straight line.

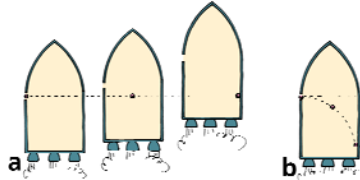




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### 16.4 General Relativity

A ball is thrown sideways in an accelerating spaceship in the absence of gravity.

a. An outside observer sees the ball travel in a straight line.  
b. To an inside observer, the ball follows a parabolic path as if in a gravitational field.

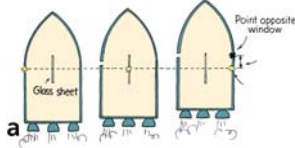




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### 16.4 General Relativity

A light ray enters the spaceship horizontally through a side window.

a. Light appears, to an outside observer, to be traveling horizontally in a straight line.

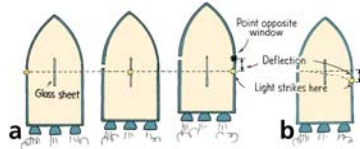




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### 16.4 General Relativity

A light ray enters the spaceship horizontally through a side window.

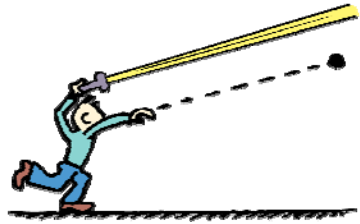

a. Light appears, to an outside observer, to be traveling horizontally in a straight line.  
b. To an inside observer, the light appears to bend.

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### 16.4 General Relativity

The trajectory of a baseball tossed at nearly the speed of light closely follows the trajectory of a light beam.

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### 16.4 General Relativity

Using his principle of equivalence, Einstein took another giant step that led him to the general theory of relativity. He reasoned that since acceleration (a space-time effect) can mimic gravity (a force), perhaps gravity is not a separate force after all.

Perhaps it is nothing but a manifestation of space-time. From this bold idea he derived the mathematics of gravity as being a result of curved space-time.

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### 16.4 General Relativity

According to Newton, tossed balls curve because of a force of gravity.

According to Einstein, tossed balls and light don't curve because of any force, but because the space-time in which they travel is curved.


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### 16.4 General Relativity

**CONCEPT CHECK:** What does the principle of equivalence state?

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### 16.5 Gravity, Space, and a New Geometry

 The presence of mass produces a curvature or warping of space-time; conversely, a curvature of space-time reveals the presence of mass.

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### 16.5 Gravity, Space, and a New Geometry

Space-time has four dimensions—three space dimensions (length, width, and height) and one time dimension (past to future).

Einstein perceived a gravitational field as a geometrical warping of four-dimensional space-time.

Four-dimensional geometry is altogether different from the three-dimensional geometry introduced by Euclid centuries earlier.

- Euclidean geometry is no longer valid when applied to objects in the presence of strong gravitational fields.

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### 16.5 Gravity, Space, and a New Geometry

#### Four-Dimensional Geometry

The rules of Euclidean geometry pertain to figures that can be drawn on a flat surface.

- The ratio of the circumference of a circle to its diameter is equal to  $\pi$ .
- All the angles in a triangle add up to  $180^\circ$ .
- The shortest distance between two points is a straight line.

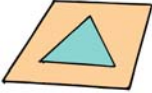
The rules of Euclidean geometry are valid in flat space, but if you draw circles or triangles on a curved surface like a sphere or a saddle-shaped object the Euclidean rules no longer hold.

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### 16.5 Gravity, Space, and a New Geometry

The sum of the angles of a triangle is not always  $180^\circ$ .

- On a flat surface, the sum is  $180^\circ$ .



The diagram shows a light blue triangle on a flat, orange square surface. The triangle's vertices are connected to the corners of the square by straight lines.

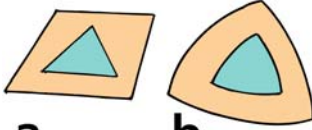
**a**

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### 16.5 Gravity, Space, and a New Geometry

The sum of the angles of a triangle is not always  $180^\circ$ .

- On a flat surface, the sum is  $180^\circ$ .
- On a spherical surface, the sum is greater than  $180^\circ$ .



The diagram shows two triangles. On the left, a light blue triangle on a flat orange square surface. On the right, a light blue triangle on a spherical orange surface, where the sides are arcs of great circles.

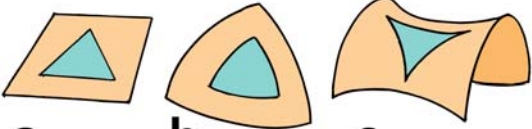
**a**      **b**

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### 16.5 Gravity, Space, and a New Geometry

The sum of the angles of a triangle is not always  $180^\circ$ .

- On a flat surface, the sum is  $180^\circ$ .
- On a spherical surface, the sum is greater than  $180^\circ$ .
- On a saddle-shaped surface, the sum is less than  $180^\circ$ .



The diagram shows three triangles. (a) A light blue triangle on a flat orange square surface. (b) A light blue triangle on a spherical orange surface. (c) A light blue triangle on a saddle-shaped orange surface.

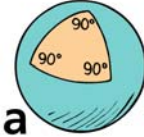
**a**      **b**      **c**

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### 16.5 Gravity, Space, and a New Geometry

The geometry of Earth's two-dimensional curved surface differs from the Euclidean geometry of a flat plane.

- The sum of the angles for an equilateral triangle (the one here has the sides equal  $\frac{1}{4}$  Earth's circumference) is greater than  $180^\circ$ .



The diagram shows a sphere with a light blue equilateral triangle on its surface. Each of the three interior angles is labeled  $90^\circ$ .

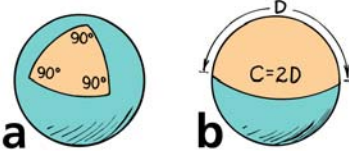
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The geometry of Earth's two-dimensional curved surface differs from the Euclidean geometry of a flat plane.

- The sum of the angles for an equilateral triangle (the one here has the sides equal  $\frac{1}{4}$  Earth's circumference) is greater than  $180^\circ$ .
- Earth's circumference is only twice its diameter instead of 3.14 times its diameter.



The diagram shows two parts. (a) A sphere with a light blue equilateral triangle on its surface, with each angle labeled  $90^\circ$ . (b) A circle with diameter  $D$  and circumference  $C = 2D$ .

**a**      **b**

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### 16.5 Gravity, Space, and a New Geometry

Of course, the lines forming triangles on curved surfaces are not "straight" from the three-dimensional view.

They are the "straightest" or *shortest* distances between two points if we are confined to the curved surface.

These lines of shortest distance are called **geodesics**.

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### 16.5 Gravity, Space, and a New Geometry

The path of a light beam follows a geodesic.

Three experimenters on Earth, Venus, and Mars measure the angles of a triangle formed by light beams traveling between them.

The light beams bend when passing the sun, resulting in the sum of the three angles being larger than  $180^\circ$ .

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### 16.5 Gravity, Space, and a New Geometry

So the three-dimensional space around the sun is positively curved.

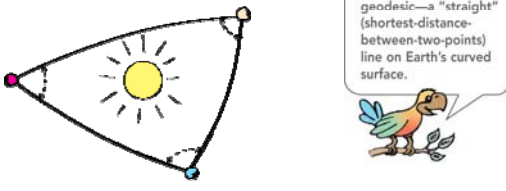
The planets that orbit the sun travel along *four*-dimensional geodesics in this positively curved space-time.

Freely falling objects, satellites, and light rays all travel along geodesics in four-dimensional space-time.

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### 16.5 Gravity, Space, and a New Geometry

The light rays joining the three planets form a triangle. Since the sun's gravity bends the light rays, the sum of the angles of the resulting triangle is greater than  $180^\circ$ .



Look at an airplane's flight path drawn on a flat map and you'll see that the line is curved. The same line drawn on the surface of a globe would be a geodesic—a "straight" (shortest-distance-between-two-points) line on Earth's curved surface.

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### 16.5 Gravity, Space, and a New Geometry

#### The Shape of the Universe

Although space-time is curved "locally" (within a solar system or within a galaxy), recent evidence shows that the universe as a whole is "flat."

There are an infinite number of possible positive curvatures to space-time, and an infinite number of possible negative curvatures, but only one condition of zero curvature.

A universe of zero or negative curvature is open-ended and extends without limit.

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### 16.5 Gravity, Space, and a New Geometry

If the universe had positive curvature, it would close in on itself.

No one knows why the universe is actually flat or nearly flat.

The leading theory is that this is the result of an incredibly large and near-instantaneous inflation that took place as part of the Big Bang some 13.7 billion years ago.

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### 16.5 Gravity, Space, and a New Geometry

General relativity calls for a new geometry: a geometry not only of curved space but of curved time as well—a geometry of curved four-dimensional space-time.

Even if the universe at large has no average curvature, there's very much curvature near massive bodies.

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### 16.5 Gravity, Space, and a New Geometry

Instead of visualizing gravitational forces between masses, we abandon altogether the idea of gravitational force and think of masses responding in their motion to the curvature or warping of the space-time they inhabit.

General relativity tells us that the bumps, depressions, and warpings of geometrical space-time *are* gravity.

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### 16.5 Gravity, Space, and a New Geometry

We cannot visualize the four-dimensional bumps and depressions in space-time because we are three-dimensional beings.

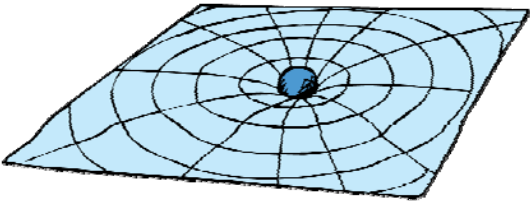
Consider a simplified analogy in two dimensions: a heavy ball resting on the middle of a waterbed.

- The more massive the ball, the more it dents or warps the two-dimensional surface.
- A marble rolled across such a surface may trace an oval curve and orbit the ball.
- The planets that orbit the sun similarly travel along four-dimensional geodesics in the warped space-time about the sun.

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### 16.5 Gravity, Space, and a New Geometry

Space-time near a star is curved in a way similar to the surface of a waterbed when a heavy ball rests on it.



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### 16.5 Gravity, Space, and a New Geometry

#### Gravitational Waves

Every object has mass, and therefore makes a bump or depression in the surrounding space-time.

When an object moves, the surrounding warp of space and time moves to readjust to the new position.

- These readjustments produce ripples in the overall geometry of space-time.

The ripples that travel outward from the gravitational sources at the speed of light are **gravitational waves**.

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### 16.5 Gravity, Space, and a New Geometry

Any accelerating object produces a gravitational wave.

In general, the more massive the object and the greater its acceleration, the stronger the resulting gravitational wave.

Even the strongest gravitational waves produced by ordinary astronomical events are the weakest kinds of waves known in nature.

Detecting gravitational waves is enormously difficult, but physicists think they may be able to do it.

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### 16.5 Gravity, Space, and a New Geometry

#### think!

Whoa! We learned previously that the pull of gravity is an interaction between masses. And we learned that light has no mass. Now we say that light can be bent by gravity. Isn't this a contradiction?

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### 16.5 Gravity, Space, and a New Geometry

**think!**

Whoa! We learned previously that the pull of gravity is an interaction between masses. And we learned that light has no mass. Now we say that light can be bent by gravity. Isn't this a contradiction?

**Answer:**

There is no contradiction when the mass-energy equivalence is understood. It's true that light is massless, but it is not "energyless." The fact that gravity deflects light is evidence that gravity pulls on the energy of light. Energy indeed is equivalent to mass!

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### 16.5 Gravity, Space, and a New Geometry

**CONCEPT CHECK:** What is the relationship between the presence of mass and the curvature of space-time?

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### 16.6 Tests of General Relativity

Upon developing the general theory of relativity, Einstein predicted that the elliptical orbits of the planets precess about the sun, starlight passing close to the sun is deflected, and gravitation causes time to slow down.


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### 16.6 Tests of General Relativity

#### Precession of the Planetary Orbits

Using four-dimensional field equations, Einstein recalculated the orbits of the planets about the sun.

- His theory gave almost the same results as Newton's law of gravity.
- The exception was that Einstein's theory predicted that the elliptical orbits of the planets should precess independent of the Newtonian influence of other planets.



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### 16.6 Tests of General Relativity

This precession would be very slight for distant planets and more pronounced close to the sun.

- Mercury is the only planet close enough to the sun for the curvature of space to produce an effect big enough to measure.

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### 16.6 Tests of General Relativity

Precession in the orbits of planets caused by perturbations of other planets was well known.

Since the early 1800s astronomers measured a precession of Mercury's orbit—about 574 seconds of arc per century. Perturbations by the other planets were found to account for the precession—except for 43 seconds of arc per century. General relativity equations applied to Mercury's orbit predict the extra 43 seconds of arc per century.



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### 16.6 Tests of General Relativity

#### Deflection of Starlight

Einstein predicted that starlight passing close to the sun would be deflected by an angle of 1.75 seconds of arc. Deflection of starlight can be observed during an eclipse of the sun.

- A photograph taken of the darkened sky around the eclipsed sun reveals the presence of the nearby bright stars.
- The positions of stars are compared with other photographs of the same part of the sky taken at night with the same telescope.

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### 16.6 Tests of General Relativity

The deflection of starlight has supported Einstein's prediction. More support is provided by "gravitational lensing," a phenomenon in which light from a distant galaxy is bent as it passes by a nearer galaxy in such a way that multiple images of the distant galaxy appear.

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### 16.6 Tests of General Relativity

Starlight bends as it grazes the sun. Point A shows the apparent position; point B shows the true position. (The deflection is exaggerated.)

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### 16.6 Tests of General Relativity

#### Gravitational Red Shift

Einstein's third prediction was that gravity causes clocks to run slow.

- Clocks on the first floor of a building should tick slightly more slowly than clocks on the top floor, which are farther from Earth and at a higher gravitation potential energy.
- If you move from a distant point down to the surface of Earth, you move in the direction that the gravitational force acts—toward lower potential energy, where clocks run more slowly.

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### 16.6 Tests of General Relativity

- From the top to the bottom of the tallest skyscraper, the difference is very small—a few millionths of a second per decade.
- At the surface of the sun compared with the surface of Earth, the clock-slowing effect is more pronounced. A clock in the deeper "potential well" at the surface of the sun should run measurably slower than a clock at Earth's surface.

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### 16.6 Tests of General Relativity

A clock at the surface of Earth runs slower than a clock farther away.

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### 16.6 Tests of General Relativity

Einstein suggested a way to measure this. Light traveling “against gravity” is observed to have a slightly lower frequency due to an effect called the **gravitational red shift**. A lowering of frequency shifts the color of the emitted light toward the red.

Although this effect is weak in the gravitational field of the sun, it is stronger in more compact stars with greater surface gravity.

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### 16.6 Tests of General Relativity

An experiment confirming Einstein’s prediction was performed in 1960 with high-frequency gamma rays sent between the top and bottom floors of a laboratory building at Harvard University.

Incredibly precise measurements confirmed the gravitational slowing of time.

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### 16.6 Tests of General Relativity

Measurements of time depend not only on relative motion, as we learned in special relativity, but also on gravity.


In special relativity, time dilation depends on the *speed* of one frame of reference relative to another one.

In general relativity, the gravitational red shift depends on the *location* of one point in a gravitational field relative to another one.

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### 16.6 Tests of General Relativity

The medieval philosopher William of Occam said that when deciding between two competing theories, choose the simpler explanation—don’t make more assumptions than are necessary when describing phenomena.



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### 16.6 Tests of General Relativity

**think!**

Why do we not notice the bending of light by gravity in our everyday environment?

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### 16.6 Tests of General Relativity

**think!**

Why do we not notice the bending of light by gravity in our everyday environment?

**Answer:**

Earth’s gravity is too weak to produce a measurable bending. Even the sun produces only a tiny deflection. It takes a whole galaxy to bend light appreciably.

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## 16.6 Tests of General Relativity

**CONCEPT CHECK:** What three predictions did Einstein make based on his general theory of relativity?

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## Assessment Questions

1. Compared to the momentum of objects moving at regular high speeds, momentum for objects traveling at relativistic speeds is
  - a. greater.
  - b. less.
  - c. the same, in accord with momentum conservation.
  - d. dependent on rest mass.

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## Assessment Questions

1. Compared to the momentum of objects moving at regular high speeds, momentum for objects traveling at relativistic speeds is
  - a. greater.
  - b. less.
  - c. the same, in accord with momentum conservation.
  - d. dependent on rest mass.

Answer: A

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## Assessment Questions

2. To say that  $E = mc^2$  is to say that energy
  - a. increases as the speed of light is squared.
  - b. is twice as much as the speed of light.
  - c. and mass are equivalent.
  - d. equals mass traveling at the speed of light squared.

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## Assessment Questions

2. To say that  $E = mc^2$  is to say that energy
  - a. increases as the speed of light is squared.
  - b. is twice as much as the speed of light.
  - c. and mass are equivalent.
  - d. equals mass traveling at the speed of light squared.

Answer: C

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## Assessment Questions

3. According to the correspondence principle,
  - a. new theory must agree with old theory where they overlap.
  - b. Newton's mechanics is as valid as Einstein's mechanics.
  - c. relativity equations apply to high speeds, while Newton's equations apply to low speeds.
  - d. special relativity and general relativity are two sides of the same coin.

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### Assessment Questions

3. According to the correspondence principle,

- new theory must agree with old theory where they overlap.
- Newton's mechanics is as valid as Einstein's mechanics.
- relativity equations apply to high speeds, while Newton's equations apply to low speeds.
- special relativity and general relativity are two sides of the same coin.

Answer: A

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### Assessment Questions

4. General relativity is most concerned with

- differences in speeds.
- differences in space-time.
- black holes.
- gravity.

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### Assessment Questions

4. General relativity is most concerned with

- differences in speeds.
- differences in space-time.
- black holes.
- gravity.

Answer: D

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### Assessment Questions

5. According to four-dimensional geometry, the angles of a triangle

- always add up to  $180^\circ$ .
- sometimes add up to  $180^\circ$ .
- never add up to  $180^\circ$ .
- only add up to  $180^\circ$  on Earth.

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### Assessment Questions

5. According to four-dimensional geometry, the angles of a triangle

- always add up to  $180^\circ$ .
- sometimes add up to  $180^\circ$ .
- never add up to  $180^\circ$ .
- only add up to  $180^\circ$  on Earth.

Answer: B

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### Assessment Questions

6. General relativity predicts that light

- becomes faster due to gravity.
- bends and clocks slow in gravitational fields.
- slows and clocks become faster in gravitational fields.
- remains unchanged throughout gravitational fields.

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### Assessment Questions

6. General relativity predicts that light
- a. becomes faster due to gravity.
  - b. bends and clocks slow in gravitational fields.
  - c. slows and clocks become faster in gravitational fields.
  - d. remains unchanged throughout gravitational fields.

Answer: B