

Chapter 3

Vectors



What is a vector

- A vector is a graphical representation of a mathematical concept
- Every vector has 2 specific quantities
 - **Magnitude**
 - length
 - **Direction**
 - angle

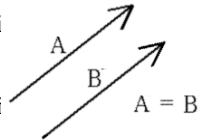
How do we draw it?



- Graphically, a vector is represented by an arrow, defining the direction, and the length of the arrow defines the vector's magnitude. This is shown above. If we denote one end of the arrow by the origin O and the tip of the arrow by Q . Then the vector may be represented algebraically by OQ .

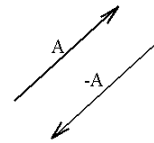
Equal Vectors

- Two vectors, A and B are equal if they have the same magnitude and direction, regardless of whether they have the same initial points, as shown in



Opposite Vectors

- A vector having the same magnitude as A but in the opposite direction to A is denoted by $-A$, as shown to the right



Properties of Vectors

- Equality of Two Vectors
 - Two vectors are **equal** if they have the same magnitude and the same direction
- Movement of vectors in a diagram
 - Any vector can be moved parallel to itself without being affected
- Resultant Vector
 - The **resultant** vector is the sum of a given set of vectors
- Equilibrium Vectors
 - Two vectors are in **Equilibrium** if they have the same magnitude but are 180° apart (opposite directions)
 - $A = -B$

Adding Vectors

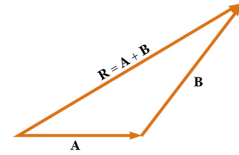
- When adding vectors, direction matters
- Units must be the same
- 2 Methods
 - **Graphical Methods**
 - Use scale drawings
 - **Algebraic Methods**
 - More convenient

Adding Vectors Graphically (Triangle or Polygon Method)

- Choose a scale
- Draw the first vector with the appropriate length and in the direction specified, with respect to a coordinate system
- Draw the next vector with the appropriate length and in the direction specified, with respect to a coordinate system whose origin is the end of vector **A** and parallel to the coordinate system used for **A**

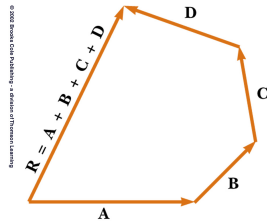
Graphically Adding Vectors, cont.

- Continue drawing the vectors "tip-to-tail"
- The resultant is drawn from the origin of **A** to the end of the last vector
- Measure the length of **R** and its angle
 - Use the scale factor to convert length to actual magnitude



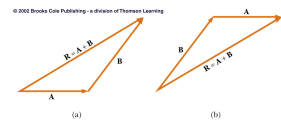
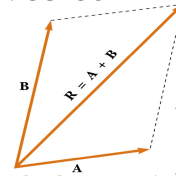
Graphically Adding Vectors, cont.

- When you have many vectors, just keep repeating the process until all are included
- The resultant is still drawn from the origin of the first vector to the end of the last vector



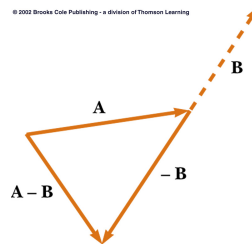
Alternative Graphical Method

- When you have only two vectors, you may use the **Parallelogram Method**
- All vectors, including the resultant, are drawn from a common origin
 - The remaining sides of the parallelogram are sketched to determine the diagonal, **R**
- Vectors obey the **Commutative Law of Addition**
 - The order in which the vectors are added doesn't affect the result



Vector Subtraction

- Special case of vector addition
- If $\mathbf{A} - \mathbf{B}$, then use $\mathbf{A} + (-\mathbf{B})$
 - $-\mathbf{B} = \mathbf{B}$ turned 180 degrees
- Continue with standard vector addition procedure

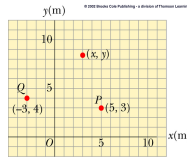


Multiplying or Dividing Vectors

- Vector * Scalar = Vector
 - Only Magnitude Change
 - What we have been doing with scaling
- Vector x Vector = Vector
 - Requires Cross Product Multiplication
 - Magnitude and Direction Change
 - Will study later in the year.

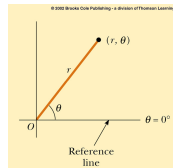
Cartesian Coordinate

- also called rectangular coordinate system
- x- and y- axes



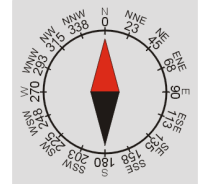
Polar Coordinates

- origin and reference line are noted
- point is distance r from the origin in the direction of angle θ , ccw from reference line
- points are labeled (r, θ)



Terrestrial (Cardinal) Coordinates

- The terrestrial coordinate system deals with directions on land.
 - 0 degrees is considered North
 - 90 degrees is considered East
 - 180 degrees is considered South
 - 270 degrees is considered West



The concept of a radian

- A radian is an arc in a circle, equal in length to the radius
- If you start with a circle, then take a line that has the same length as the radius of that circle and bend it around the circle's circumference, it will encompass an angle of one radian.

Radian vs degree

- There are 360 degrees in a circle
- There are 2π radians in a circle
- Therefore 1 radian = $360 \text{ degree} / (2\pi)$
 - This value is equal to 57.3 degrees
- The conversion factor is then 1 rad = 57.3 degrees

Great Greeks

- **Hipparchus of Nicaea, (190-120 B.C.)** who was born about 190 B.C., was a creative and talented Greek astronomer and mathematician. **He founded trigonometry** and scientific geography. He discovered precession of equinoxes and invented methods of fixing terrestrial positions by circles of latitude and longitude. He located 850 stars and divided the stars that he could see into classes of apparent brightness. Hipparchus estimated the size and distance of the moon, found a way to predict eclipses, as well as calculating the length of the year to within 6 and a 1/2 minutes!

Trigonometry

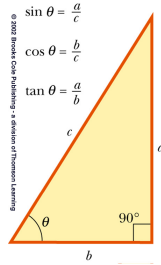
- Originally TRIGONOMETRY was that branch of mathematics concerned with solving triangles using trigonometric ratios which were seen as properties of triangles rather than of angles.
- The word Trigonometry comes from the Greek words : **Treis = three, Gonias = angle and Metron = measure.** The Early Greeks developed the subject by studying the relationship between the arc of the circle - the measure of the central angle - and the chord of the arc.

SOH CAH TOA

$$\sin(\theta) = \frac{Opp}{Hyp}$$

$$\cos(\theta) = \frac{Adj}{Hyp}$$

$$\tan(\theta) = \frac{Opp}{Adj}$$



Common angles

- We use calculators to get the correct ratios
- Calculators have built into them a logarithmic table for these comparisons
- Some common ones we will run into are
 - The sine (0) = 0
 - The sine (90) = 1
 - The cosine (0) = 1
 - The cosine (90) = 0
 - The tangent (0) = 0
 - The tangent (45) = 1
 - The tangent (90) = D.N.E.

More Trigonometry

- To find an angle, you need the inverse trig function
 - for example,

$$\theta = \sin^{-1} 0.707 = 45^\circ$$

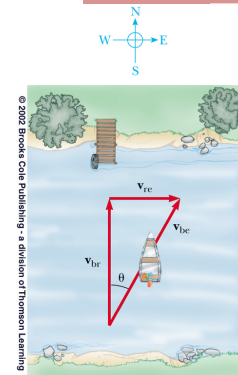
- Inversing "frees theta"

Solve

■ $V_{br} = 10\text{m/s}$

■ $V_{rc} = 3\text{m/s}$

■ What is V_{bc} ?

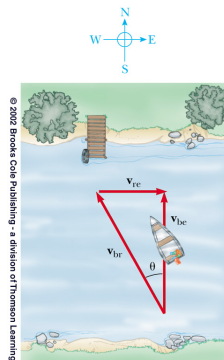


Solve

■ $V_{br} = 10\text{m/s}$

■ $V_{rc} = 3\text{m/s}$

■ What is theta have to be for the boat to go straight across?



Great Greeks

- **Pythagoras of Samos (569-500 B.C.)** was a Greek philosopher and mathematician, whose doctrines were strongly influenced by Plato. Among the extensive mathematical investigations carried on by the Pythagoreans were their studies of odd and even numbers and of prime and square numbers. From this arithmetical standpoint they cultivated the concept of number, which became for them the ultimate principle of all proportion, order, and harmony in the universe. Through such studies they established a scientific foundation for mathematics.

Great Greeks

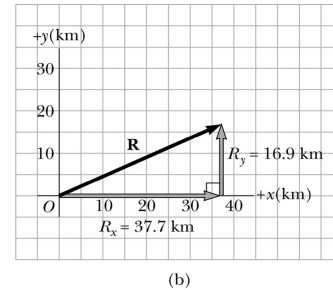
- Pythagoras Continued**
 In geometry the great discovery of the school was the hypotenuse theorem, or Pythagorean theorem, which states that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides. *Pythagorean theorem* which was however known much earlier (From the Babylonians and perhaps earlier from the Chinese). Platonic solids are perfectly regular solids with the following conditions: all sides are equal and all angles are the same and all faces are identical. In each corner of such a solid the same number of surfaces collide. Only five Platonic solids exist: tetrahedron, hexahedron, octahedron, dodecahedron and icosahedrons.

- <http://www.3quarks.com/GIF-Animations/PlatonicSolids/>
- http://usuarios.advanco.com.ar/simetriadelospacio/symmetry_space/orbits_venus_jupiter.htm

More Trigonometry

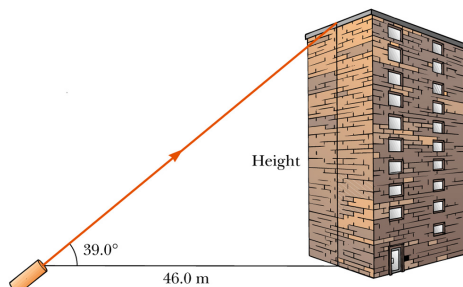
- Pythagorean Theorem**

$$c^2 = a^2 + b^2$$



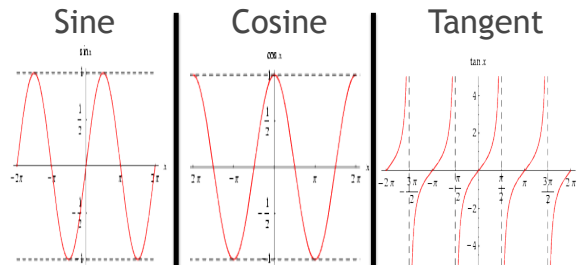
© 2002 Brooks Cole Publishing - a division of Thomson Learning

What is the height



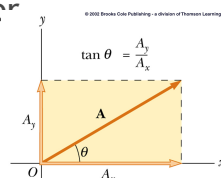
© 2002 Brooks Cole Publishing - a division of Thomson Learning

Graphs



Components of a Vector

- A component** is a part
- Rectangular components**
 - These are the projections of the vector along the x- and y-axes



- The x-component of a vector, projects along the x-axis
 $A_x = A \cos \theta$
- The y-component of a vector, projects along the y-axis
 $A_y = A \sin \theta$
- Then,

$$\mathbf{A} = A_x + A_y$$

More About Components of a Vector

- The previous equations are valid **only if θ is measured with respect to the x-axis**
- The components can be positive or negative and will have the same units as the original vector
- The components are the legs of the right triangle whose hypotenuse is **A**

$$A = \sqrt{A_x^2 + A_y^2} \quad \text{and} \quad \theta = \tan^{-1} \frac{A_y}{A_x}$$

- May still have to find θ with respect to the positive x-axis

Vector Notation

- Vector notation allows us to treat the components separate in an equation. Just like you wouldn't add together $3x+4y$ because they are different variables.
- The coordinates (a,b,c) it can be expressed as the sum of three vectors $a\hat{i} + b\hat{j} + c\hat{k}$

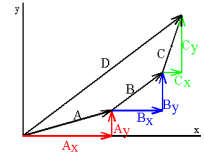
- Example

$$\vec{A} = 3\hat{i} - 5\hat{j} \quad \vec{B} = -2\hat{i} + 7\hat{j}$$

Adding Vectors Algebraically

1. Convert to polar coordinates if not already in Cartesian coordinates and sketch the vectors
2. Find the x- and y-components of all the vectors
3. Add all the x-components
 - This gives R_x :

$$R_x = \sum v_x$$



Adding Vectors Algebraically, cont.

4. Add all the y-components
 - This gives R_y : $R_y = \sum v_y$
5. Use the Pythagorean Theorem to find the magnitude of the Resultant: $R = \sqrt{R_x^2 + R_y^2}$
6. Use the inverse tangent function to find the direction of R:

$$\theta = \tan^{-1} \frac{R_y}{R_x}$$

Adding Vectors Algebraically, HARDEST PART

7. Put your angle in the correct quadrant

$$\theta = \text{atan} \frac{R_y}{R_x} + 180^\circ \quad \begin{array}{|c|c|} \hline \text{Quadrant} & \\ \hline \text{II} & \text{I} \\ \hline \text{III} & \text{IV} \\ \hline \end{array} \quad \theta = \text{atan} \frac{R_y}{R_x} \quad \begin{array}{l} \text{gives a negative angle in quadrant IV} \\ \text{Can add } 360^\circ \text{ to get standard angle.} \end{array}$$

Adding Vectors Example #2

$$\vec{R}_1 = 4\hat{i} - 6\hat{j} \quad \vec{R}_2 = [11, 160^\circ]$$

Converting between Coordinate Systems

- Polar to rectangular
 - <http://www.univie.ac.at/future.media/moe/galerie/zeich/zeich.html>

Converting between Coordinate Systems

Terrestrial (Cardinal) Coordinates to Polar

- The terrestrial coordinate system deals with directions on land.
 - 0 degrees C.C. is 90 degrees P.C.
 - 90 degrees C.C. is 0 degrees P.C.
 - 180 degrees C.C. is 270 degrees P.C.
 - 270 degrees C.C. is 180 degrees P.C.
- It is best to draw a picture

Converting between Coordinate Systems

Terrestrial (Cardinal) Coordinates to Polar

- These coordinates are broken into sub coordinates
 - 20 degrees North of East = 20 degrees p.c.
 - 20 degrees East of North = 70 degrees p.c.
 - Note that you start in the second direction and go in the direction of the first.
 - So if you are going west of north, you start in the north and go west