Lab 1
Standing Waves on a String

Learning Goals:

- To distinguish between traveling and standing waves
- To recognize how the wavelength of a standing wave is measured
- To recognize the necessary conditions to establish a standing wave
- To see and hear the standing wave established on a stretched string or in a volume of air.
- To observe a **standing** transverse mechanical **wave** on a stretched string of total length Ł and note its shape, length and amplitude.
- To measure the wave's segment (L) noting there are an integral number of segments each of equal length L along the string of total length Ł when a standing wave is being generated.
- To measure the string's tension (F) and its mass per unit length (d).
- To compute the length (λ) of the standing wave from the length of its segment (L).
- To compute the experimental value of the velocity (v) of the standing wave from its frequency (f) and its wavelength (λ).
- To compute the theoretical value of the velocity of a mechanical wave determined by the tension and mass per unit length of the elastic medium (string) through which it travels.
- To compare the experimental value of the frequency of the standing wave on a sting under changing tension with the frequency of a vibrator (120 Hz.) generating the standing wave.
- To investigate the relationship between the frequency of vibration, and the tension, density and length of a stretched string and the production of a standing wave vibrating along the string.
- To predict the relationship between the fundamental frequency and the frequencies that produce standing wave patterns and their lengths.

Apparatus:

Gram weights with weight hanger, pulley, meter stick, two speakers

INSTRUMENT	INSTRUMENTAL ERROR (of full scale)	INSTRUMENTAL RESOLUTION
Computer with screen and speaker	Tutorial simulation	
Data Studio software CA-6787, 49 Wave Properties.ds.	Tutorial simulation	
String vibrator	F = 120 Hz @ ± 1.0 %	0.1 Hz
Frequency Generators	Volts/seconds	See OSCILLOSCOPE
String of linear density μ =______ kg/m	± 0.10%	0.0001 kg/m
Triple Beam Balance	± 10% (M < 1g), ± 5% (1g < M < 10g), ± 1% (10 g <M < 100 g)± 0.1% (100 g < M < 1000 g)	0.1 gram

Theory:

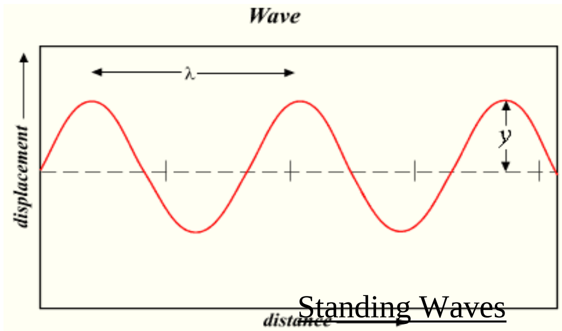
A mathematical interpretation of a wave is called a *wave equation*. For many types of waves (sound waves, water waves, string waves), the wave equation represents the *displacement* of the *medium* through which the wave moves (air for sound waves, water surface for water waves, string for string waves, etc.). Some waves, such as electromagnetic waves, don't require a medium. In that case, the wave equation represents something else. However, the general basic form of the wave equation is

(1) $\mathbf{y}(x,t) = \mathbf{A} \sin(\mathbf{k}x - \boldsymbol{\omega}t + \boldsymbol{\varphi_0})$

$\mathbf{y}$ ≡ "displacement of medium"
$\mathbf{A}$ ≡ "amplitude"
$\mathbf{k}$ ≡ "wave number" = $2\pi / \boldsymbol{\lambda}$

$\boldsymbol{\omega} \equiv$ “angular frequency” = 2π / **T** = 2π**f**
$\boldsymbol{\varphi_0} \equiv$ “phase constant”

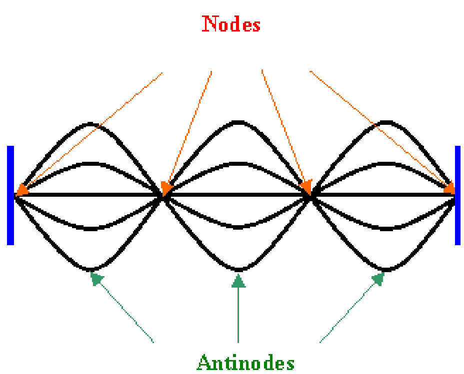
For any wave, **λ** (lambda) is the *wavelength* (the distance between two sequential crests or troughs of the wave). **T** is the *period* of oscillation (number of seconds per cycle), and **f** is the *frequency* (number of cycles per second). The figure below shows the wave function graphed as displacement vs. distance (this is for t = 0, a “snapshot” graph).

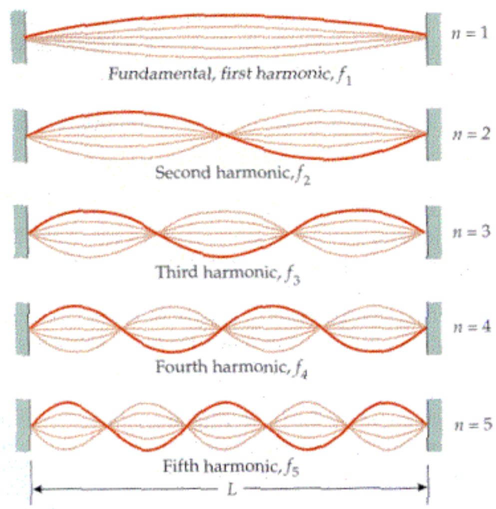


Standing Waves

A special case arises when two waves travelling in opposite directions overlap. These two waves will undergo *wave interference*, which will create a new waveform from the initial two. If those two waves have the same frequency and wavelength, a standing wave will occur. This special type of wave will appear not to move, but will oscillate in place. Some points will appear not to displace at all (called *nodes*), while other points will appear to displace more than any points around them (called *antinodes*).

For standing waves on a string, the number of antinodes for a particular vibration is called the string’s *mode*, or *harmonic*. For instance, two antinodes would represent a string’s <u>second</u> harmonic (see figure below).





When a stretched string is plucked it will vibrate in its fundamental mode in a single segment with nodes on each end. If the string is driven at this fundamental frequency, a standing wave is formed. Standing waves also form if the string is driven at any integer multiple of the fundamental frequency. These higher frequencies are called the harmonics.

Each segment is equal to half a wavelength. In general for a given harmonic, the wavelength is shown by (EQ1)
$\lambda = 2L/n$ where **L** is the length of the stretched string and **n** is the number of segments in the string.

The linear mass density of the string can be directly measured by weighing a known length of the string: **μ = mass / length**

The linear mass density of the string can also be found by studying the relationship between the tension, frequency, length of the string, and the number of segments in the standing wave. To derive this relationship, the velocity of the wave is expressed in two ways.

The velocity of any wave is given by where f is the frequency of the wave. For a stretched string:

For all waves:

(EQ 2) **v = λf**

v ≡ "wave velocity"

λ ≡ "wavelength"

f ≡ "frequency"

Combing (EQ1) and (EQ2) we get

(EQ3) v = 2Lf/n

When dealing with the velocity of a wave traveling in a string is also dependent on the tension, **T**, in the string and the linear mass density, **μ**, of the string:

(EQ4) **v** = √(**T**/**μ**)

v ≡ "wave velocity"

T ≡ "tension on string"

μ ≡ "linear density" = mass/length

Note: (EQ4) is derived as a solution to the second derivative of the one-dimensional wave equation by direct substitution. Students in Phys 141 should look at http://hyperphysics.phy-astr.gsu.edu/hbase/waves/wavsol.html#c2

(EQ2) and (EQ4) can be set equal to each other, which can be rewritten as **λf =** √(**T**/**μ**). Solving for **T**, the result is

(EQ5) $T = \left(4L^2 f^2 \mu\right)\left(\frac{1}{n^2}\right)$

Solving (EQ5) for f:

(EQ6) $f = \sqrt{\frac{T}{4L^2 \mu}}n$

Solving for λ we get

(EQ7) λ = (1/**f**√**μ**) √**T**

Where (EQ7) is a linear equation (with respect to $\sqrt{\mathbf{T}}$), in the form y=mx+b where b=0 so:

(EQ8) Slope = $(1/\mathbf{f}\sqrt{\boldsymbol{\mu}})$

Prelab:

Part I Simulating Transverse, Traveling Waves

1. Perform the following activity
   a. On the line below use a ruler to make dots at 0, 1, 2, 3, …10 cm. and label the line "centimeters"
   b. Draw a sine wave so that the wave intersects the ruler points on the line and has a height and depth 1.5 cm above and below the line. This curve represents a "snapshot" of a traveling, transverse, mechanical wave on a stretched string.
   c. Use your drawing to answer questions 2-5 below.

____.

2. What is the amplitude of the wave**?** __________cm
3. What is the wavelength of the wave? __________cm.
4. If the frequency of the wave is 10 Hz, what is the velocity of this traveling wave? ______________cm/sec.
5. What equation describes the vertical displacement, D in meters, of this traveling, transverse wave?

   Ans.: $D = 0.015[\sin (2\pi/0.02)x + (10)2\pi t] = 0.015[\sin 100\pi x + 20\pi t]$

Part II: Simulating Standing Transverse Waves

1. Perform the following activity
   a. On the line below use a ruler to make dots at 0, 1, 2, 3, …10 cm. and label the line "centimeters"
   b. On the line draw a sine wave so that the wave intersects the ruler points on the line and has a height and depth 1.5 cm above and below the line. This curve represents a "snapshot of a traveling, transverse, mechanical wave on a stretched string.
   c. On the line draw another sine wave so that the wave intersects the ruler points on the line and has a height and depth 1.5 cm above and below the line. This curve represents a "snapshot" of another traveling, transverse, mechanical wave on a stretched string .
   d. On the line draw the addition of the amplitudes of these two sine waves with a pen of a different color. This addition represents a snapshot of a standing wave on a stretched string which results when two traveling waves of the same length cross through each other on the same string at the instant when they are zero degrees out of phase (i.e., in phase) with each other.
   e. Use your drawing to answer questions 2-6 below.

____.

2. What is the maximum amplitude of the standing wave**?** __________cm
3. What is the length of one segment of this standing wave? ______cm
4. What is the wavelength of the standing wave? __________cm.
5. If the frequency of the standing wave is 10 Hz, what is the velocity of this standing wave? __________cm/sec.

6. What equation describes the vertical displacement, D in meters, of this standing, transverse wave?

   Ans.: $D = 0.030[\sin(2\pi/0.02)x][\cos(10)2\pi t] = 0.030[\sin 100\pi x][\cos 20\pi t]$

Procedures 1:

For this lab, a string (with linear density ≡ $\boldsymbol{\mu} = 2.4 \times 10^{-4}$ Kg/m) is bound between a vibrating motor (with known frequency ≡ $\mathbf{f_{actual}}$ = 120 Hz) and a pulley which supports a hanging mass at the other end of the string. This mass applies a tension force ≡ **T** to the string.

If mass is the string density is not know complete the following procedure:

Direct Calculation of the Linear Mass Density

1. Measure the mass of a known length (about 20 m) of the string.

   Length = L = _______________ Meters

   Mass = M = ________________ Kilograms

2. Calculate the linear mass density by dividing the mass by the length (μ = Mass/Length): Record this value on the line below

   Linear Density = ρ = ________________ Kilograms / Meter

Procedure 2: Generating Standing Waves of Fixed Frequency on a Stretched String Under Changing Tension.

1. Attach the string to an electric vibrator of known, fixed frequency f = 120 Hertz.
2. Attach a weight hanger to the string and suspend it over the pulley. Insert weights that are sufficient to establish a tension in the string that produces a standing wave when the vibrator is on.
3. Adjust the weight for the maximum amplitude of the standing wave. The length of the string can also be adjusted to effect maximum amplitude
4. Carefully measure (from the top of the pulley) the length of one vibrating segment l of the standing wave and record its length in meters in the data table. The length of a segment is from the top of the pulley to the next node. A node is a point of zero amplitude.
5. Calculate the length of the standing wave by multiplying the segment length by two and record this wavelength in the data table. ($\lambda = 2l$).
6. Repeat procedures #2 to #5 adjusting the tension in the string with weights until eight different modes are measured. The mode number is the number of segments in the standing wave pattern along the entire length of the stretched string. (The tension to produce the mode 1 wave is very large; the string may break; therefore it may be omitted) (The tension to produce the mode 8 wave is very small and its amplitude may be difficult to see; therefore, mode 8 may be omitted). Record the data in the data table.
7. Graph the wavelength of the standing wave (ordinates) against the square root of the tension (abscissas). (Note you may using graphing program like Excel.) Connect the data in the best fit straight line and determine its slope.
8. Calculate the experimental value of the frequency of the standing wave modes from equation number (**7**). Include the graph in the notebook and complete the second table.
9. Answer the questions after completing the data tables.

Data Table Procedure 2 Fixed frequency (record in notebook)

Density of string μ = ________ kg/m

Mode	One segment	Wave length	Mass m stretching	Tension mg in	Square root of Tension

# n	length l (meters)	λ (meters)	String (kg)	string (nt)	$\sqrt{}$(mg) in string (nt)$^{1/2}$
1	optional				
2					
3					
4					
5					
6					
7					
8	(optional)				

Accepted value of frequency (Hz)	Experimental value of frequency (Hz)	% difference	Experimental Velocity of the standing wave in second mode (n = 2) using equation (**5**) (m/sec)	Experimental value of velocity of waves forming the standing wave in the second mode (n = 2) using equation (**3**) (m/sec)	% difference
120.0					

Questions Part II:

1) What factors in this lab affected the accuracy of $\mathbf{f}_{\mathbf{experimental}}$?

2) a) Sketch a standing string wave vibrating in its fourth mode. Identify and label the nodes, antinodes, wavelength and amplitude in your sketch.

b) If the velocity of a wave on this string is 10 m/s, find the *expression* of the wave's frequency in terms of its wavelength.

3) For a standing string wave with a wavelength of 1 meter in its fifth mode, what is the *value* for the total length [meters] of the vibrating string?

4) Suppose a wave is traveling along a string with a given tension and linear density. Write the *expression* for the new wave velocity of the string wave when:
(Please express your answers as a multiple of the original wave velocity)

a) the tension is doubled?

b) the tension is tripled?

c) the tension is halved?

d) the tension is multiplied by n?

e) the linear density is doubled?

5) For any wave with a wavelength $\boldsymbol{\lambda}$ = 500 nm, a frequency **f** = 600 THz, and an amplitude **A** = 10. Find the *value* for the:

a) wave number ≡ **k**.

b) angular frequency ≡ **ω**.

c) wave velocity ≡ **v**.

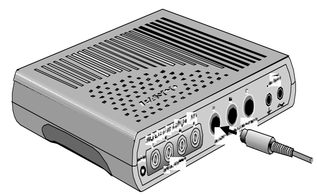
d) What type of wave is this? (Hint: Does the value for wave velocity look familiar?)

e) Write the wave equation for this wave using your answers from above. Use equation (1) as the general form, with phase constant $\varphi_0 = 0$.

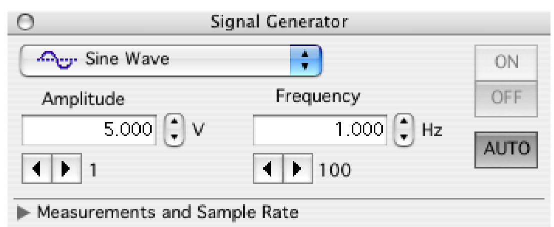
f) What is the *value* of displacement **y** when x = 4.5 meters, t = 0 seconds?

Procedure III: Standing waves of variable frequency on a stretched string.

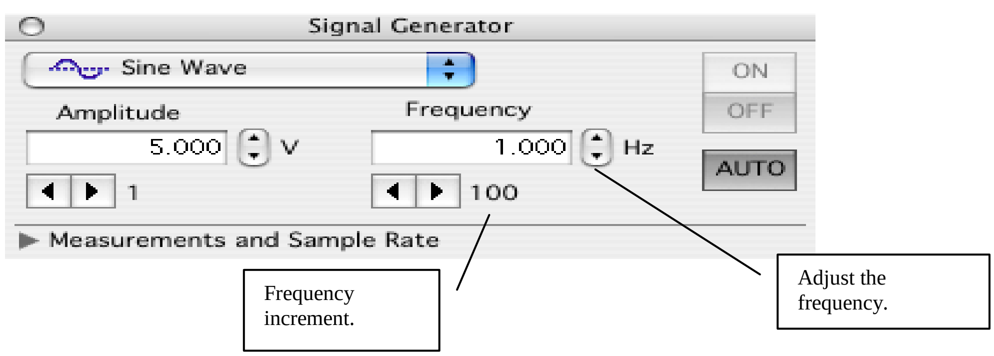
1. Arrange the equipment as shown below in the diagram of the apparatus.
2. Set up the PASCO 750 Interface and computer and start Data Studio.



3. Connect the Power Amplifier to Analog Channel A. Connect the power cord into the back of the Power Amplifier and plug it in.
4. Open the *DataStudio* file: **50 Standing Waves.ds.** The *DataStudio* file has a Signal Generator window that controls the output of the Power Amplifier. Attach an oscilloscope to observe the amplitude and period of the electrical wave controlling the vibrator.



5. Clamp the Variable String Vibrator and the pulley about 1.2 m apart. Tie one end of the same density string μ to the Variable String Vibrator. Put the string over the pulley and hang about 0.150 kg (150 g) from it <u>including the weight hanger.</u> The tension in the string mg = 1.47 nt.
6. Use two banana plug patch cords to connect the SIGNAL OUTPUT of the Power Amplifier to the String Vibrator.



7. Click ‘Start’ in *DataStudio*. The output from the Power Amplifier starts automatically.
8. Change the frequency increment in the Signal Generator to ‘1’ and adjust the frequency so that the string **vibrates in one segment** n = 1. Adjust the amplitude and frequency to obtain a large-amplitude wave. Check the end of the string over the pulley: this is a node; check the end of the vibrator where the string attaches to it: this is an anti-node.
9. Record the fundamental frequency as f_1 for mode n = 1 in the Data section

10. Measure the distance from where the string attaches to the vibrator to the top of the pulley as L. Note that L is not the total length of the string, only the part of the string that vibrates. This is mode #1. Record in the data table this distance as L in the Data Table
11. Use the equation for wave speed to calculate the wave speed of the one-segment standing wave. Record the wave speed in the notebook.
12. If a strobe light is available, use it to illuminate the vibrating string. Adjust the strobe's frequency so it matches the String Vibrator's frequency.
13. Adjust the frequency so the standing wave pattern has *two* segments (two antinodes with a node in the center). Record the new frequency f_2 in the Data Table.
14. Calculate the ratio of the two-segment frequency to the fundamental frequency and record the ratio in the data table in the notebook.
15. Calculate the wave speed of the two-segment standing wave and record the calculation in the data table in the notebook.
16. Adjust the frequency so the string vibrates in *three* segments (three antinodes). Record the frequency f_3 in the Data table in the notebook. Calculate the ratio of the three-segment frequency to the fundamental frequency and record the ratio in the Data table in the notebook.
17. Calculate the wave speed of the three-segment wave and record the calculation in the data table. Click 'Stop' to end data recording.
18. Use the same setup as in procedures t - w, but hang about 0.050 kg (50 g) from the string over the pulley. Measure the distance from the end of the vibrator to the top of the pulley and record this as L in the data table.
19. Click 'Start'. Use the Signal Generator window to adjust the frequency of the Power Amplifier so the string vibrates in one segment. This is mode #1 for a smaller wave velocity than in procedure w because the tension in the string has been decreased (from 150 g to 50 g). Record the tension and the frequency f_1 for this mode #1 standing wave in the Data table in the notebook.
20. Using the 50 gram tension, click 'Start'. Use the Signal Generator window to adjust the frequency of the Power Amplifier so the string vibrates with *four* segments. Record the frequency f_4 in the Part IIB Data table. Record the tension in the string and measure the wavelength of this mode #4 standing wave and record it in the Data Table in the notebook.
21. Increase the tension in the string by adding 50 grams and repeat step 20. Measure and record the tension; adjust and record the frequency f_4 in the Data table.
22. Continue to add increments of 50 g to the hanging mass up to 250 g. repeating procedure 20. Click 'Stop.' Record this data in the data table and calculate then record there the square of the frequency.
23. Make a graph of frequency *squared* (f_4^2) on the y-axis versus tension in string, *mg,* on the x-axis. Use Hz^2 as the unit for frequency squared and newtons as the unit for tension.
24. Find the slope of the best fit line for the data. The slope indicates the experimental value of the density of the string (using equation #6) $\mu = 1/(\textbf{slope}\lambda^2)$ Record the slope and tape the graph in your notebook..
25. Using the slope of the graph, calculate the experimental value of μ. The unit is kg/m. Record the experimental value of μ for the string in the data table in the notebook.
26. Compare the given linear density with the linear density determined from the graph by computing the percent difference.
27. Answer the questions at the end of the data tables and record these in notebook.
28. Compare the experimental and given values of the string's density using percent difference and record the result in the data table in the notebook

Data Table Procedure 3 (record in notebook)

7. a) String of constant length and uniform μ= ____________kg/m

String length L (m)	Number of segments n	Tension in string mg (newton)	Length of one segment *l* (meters)	Length of standing wave λ (meters)	Frequency (Hertz)	Velocity of standing wave v (m/sec)
	one	1.47			f_1 =	
As above	two	1.47			f_2 =	

As above	three	1.47			$f_3 =$	
As above	one	0.490			$f_1 =$	

b) Ratio of two segment frequency to the fundamental frequency: ____.
c) Ratio of three segment frequency to the fundamental frequency: ____.
d) Data: ***four*** segment standing wave on string of constant length L _____m and uniform μ=____kg/m.

Total hanging Mass (grams)	50	100	150	200	250
Tension in string mg (newtons)					
Frequency f_4 (Hz)					
Square of Frequency f_4^2 (Hz^2)					

e) Slope of graph of square of frequency vs tension = ____________.
f) Experimental value of linear density from slope (8) μ = _____kg/m.
g) Compose a general statement that describes how the frequency of vibration of a standing wave relates to the number of segments in the pattern of the standing wave: ____________.
h) Compose a general statement that describes how the velocity of the standing wave depends on the tension______________________.
i) Describe the relationship between the fundamental frequency and the frequencies that produce standing wave patterns. ___________.
j) For a stretched string of constant length and tension, when the frequency changes, the length of the standing wave changes and the mode pattern changes. Describe the relationship between the values of frequency, wavelength and mode.____________.