Cross Product Notes

The Vector Product – Vector multiplication of two vectors can give a product that is also a vector. A vector product has both **magnitude** and **direction**. Before stating the complete definition of the vector product, we will discuss notation and a rule about direction.

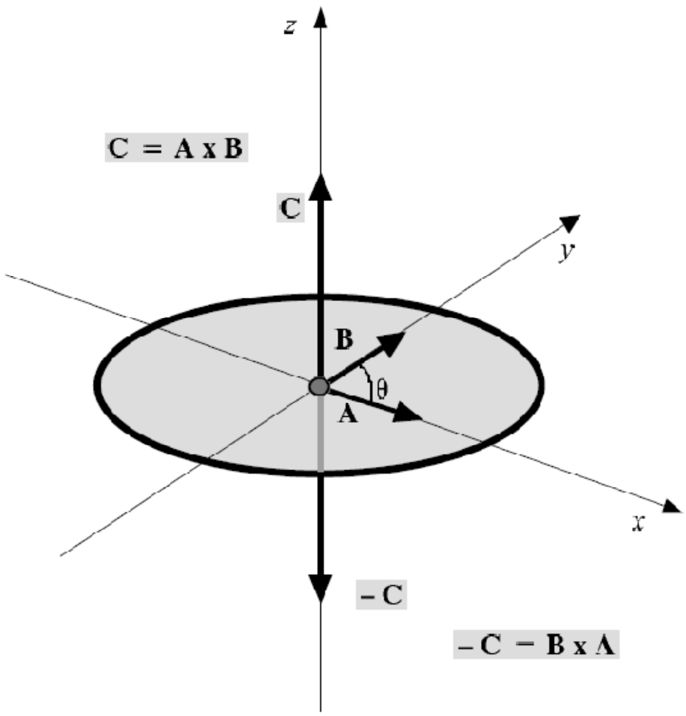
The vector product of **A** and **B** is written as **A x B** and is read as "*A* **cross** *B*". The cross distinguish from a scalar, or **dot**, product. The vector **C** is the cross product of A and B.

In the figure below, the result of **A x B** is C, which is perpendicular to the plane formed by A and **B**. this case, **C** points along the *z*-axis, and upward. But, if we had multiplied **B x A** (as was done in the lower portion of the drawing), the product C would be downward along the *z*-axis.

There is a simple rule that gives the direction of C. With the right hand opened flat, perpendicular to the palm facing the origin of the graph, and the fingers extending toward **B**, imagine that you are rotating A toward **B**. You notice the thumb points upward, the direction of C.

Now reorient your hand perpendicular to **B**, palm toward origin, and fingers extended toward **A**. Ima that you are moving from **B** to A. You have had to turn your hand upside down and the thumb points downward. The rule is often called the **RIGHT-HAND** rule.

The vector product of two vectors A and **B** is a vector C whose magnitude is the product of their magnitudes times the *sine* of the angle between the two vectors and whose direction, perpendicular to A and given by the **right-hand** rule. Hence, the vector product is written:



$\mathbf{C} = \mathbf{A} \times \mathbf{B}$, where its magnitud given by $C = \mathbf{AB} \sin \theta$.

Conversely, the vector – C is written as follows:

$\mathbf{B} \times \mathbf{A} = -\mathbf{C}$, where its magnitude is given by $-C = \mathbf{BA} \sin \theta$. Both C and – **C** are shown in the figure at le

C is said to be *orthogonal* to both **A** and **B**. **Orthogonal** is similar to the word *perpendicular*, but as perpendicular refers to two lines at a right angle to each other within the *same* plane, orthogonal means at a right angle ***to*** another plane.

C is orthogonal to the plane that contains **A** and **B**.

Some important properties of the vector product which follow from its definition are as follows:

1. Unlike the scalar product, the order in which the two vectors are multiplied in a cross product is important, that is:

$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$$

Therefore, if you change the order of the cross product, you must change the sign. You could easily verify this relation with the right-hand rule.

2. If **A** is parallel to **B** (θ = 0 ° or 180 °), then **A x B** = 0; therefore, it follows that **A x A** = 0.

3. If **A** is perpendicular to **B**, then **A x B** = AB.

4. It is also important to note that the vector product obeys the **distributive law**, that is,

$$\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{C}$$

5. Cross products of the rectangular unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ obey the following expressions:

$$\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$$

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = -\hat{\mathbf{j}} \times \hat{\mathbf{i}} = \hat{\mathbf{k}}$$

$$\hat{\mathbf{j}} \times \hat{\mathbf{k}} = -\hat{\mathbf{k}} \times \hat{\mathbf{j}} = \hat{\mathbf{i}}$$

$$\hat{\mathbf{k}} \times \hat{\mathbf{i}} = -\hat{\mathbf{i}} \times \hat{\mathbf{k}} = \hat{\mathbf{j}}$$

6. Signs are interchangeable. For example, $\hat{\mathbf{i}} \times -(\hat{\mathbf{j}}) = -\hat{\mathbf{i}} \times \hat{\mathbf{j}} = -\hat{\mathbf{k}}$.

7. The cross product of *any* two vectors **A** and **B** can be expressed in the following **matrix determinant** form:

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Expanding this determinant gives the following result:

$$\mathbf{A} \times \mathbf{B} = (A_yB_z - A_zB_y)\hat{\mathbf{i}} + (A_zB_x - A_xB_z)\hat{\mathbf{j}} + (A_xB_y - A_yB_x)\hat{\mathbf{k}}$$

Some important physical quantities are expressed as vector products. The torque **L** (also called the moment) of a force **F** with the moment arm **r** is given by: **L = r x F**.

Also, the energy carried by radio waves travels in the direction of the cross product of the **electric** and **magnetic field** vectors.