

## Coordinate Systems

### Cartesian Coordinate System

Excerpt from [http://en.wikipedia.org/wiki/Cartesian\\_coordinate\\_system](http://en.wikipedia.org/wiki/Cartesian_coordinate_system)

A Cartesian coordinate system specifies each point uniquely in a plane by a pair of numerical coordinates, which are the signed distances from the point to two fixed perpendicular directed lines, measured in the same unit of length. Each reference line is called a coordinate axis or just axis of the system, and the point where they meet is its origin. The coordinates can also be defined as the positions of the perpendicular projections of the point onto the two axes, expressed as a signed distances from the origin.

One can use the same principle to specify the position of any point in three-dimensional space by three Cartesian coordinates, its signed distances to three mutually perpendicular planes (or, equivalently, by its perpendicular projection onto three mutually perpendicular lines). In general, one can specify a point in spaces of any dimension  $n$  by use of  $n$  Cartesian coordinates, the signed distances from  $n$  mutually perpendicular hyperplanes.

The invention of Cartesian coordinates in the 17th century by René Descartes revolutionized mathematics by providing the first systematic link between Euclidean geometry and algebra. Using the Cartesian coordinate system, geometric shapes (such as curves) can be described by Cartesian equations — algebraic equations involving the coordinates of the points lying on the shape. For example, the circle of radius 2 may be described as the set of all points whose coordinates  $x$  and  $y$  satisfy the equation  $x^2 + y^2 = 2^2$ .

Cartesian coordinates are the foundation of analytic geometry, and provide enlightening geometric interpretations for many other branches of mathematics, such as linear algebra, complex analysis, differential geometry, multivariate calculus, group theory, and more. A familiar example is the concept of the graph of a function. Cartesian coordinates are also essential tools for most applied disciplines that deal with geometry, including astronomy, physics, engineering, and many more. They are the most common coordinate system used in computer graphics, computer-aided geometric design, and other geometry-related data processing.

### History

The adjective *Cartesian* refers to the French mathematician and philosopher René Descartes (who used the name *Cartesius* in Latin).

The idea of this system was developed in 1637 in two writings by Descartes and independently by Pierre de Fermat, although Fermat used 3 dimensions and did not publish the discovery. In part two of his *Discourse on Method*, Descartes introduces the new idea of specifying the position of a point or object on a surface, using two intersecting axes as measuring guides. In *La Géométrie*, he further explores the above-mentioned concepts.

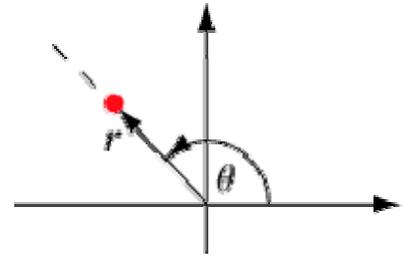
It may be interesting to note that some have indicated that the master artists of the Renaissance used a grid, in the form of a wire mesh, as a tool for breaking up the component parts of their subjects they painted. That this may have influenced Descartes is merely speculative. Representing a vector in the standard basis. The development of the Cartesian coordinate system enabled the development of calculus by Isaac Newton and Gottfried Wilhelm Leibniz.

Many other coordinate systems have been developed since Descartes, such as the polar coordinates for the plane, and the spherical and cylindrical coordinates for three-dimensional space.

Read more facts at [http://en.wikipedia.org/wiki/Cartesian\\_coordinate\\_system](http://en.wikipedia.org/wiki/Cartesian_coordinate_system)

### Polar Coordinate System

In mathematics, the polar coordinate system is a two-dimensional coordinate system in which each point on a plane is determined by a distance from a fixed point and an angle from a fixed direction. The fixed point (analogous to the origin of a Cartesian system) is called the pole, and the ray from the pole with the fixed direction is the polar axis. The distance from the pole is called the radial coordinate or radius, and the angle is the angular coordinate, polar angle, or azimuth.



The polar coordinates  $r$  (the radial coordinate) and  $\theta$  (the angular coordinate, often called the polar angle) are defined in terms of Cartesian coordinates by

$$x = r \cos \theta \quad (1)$$

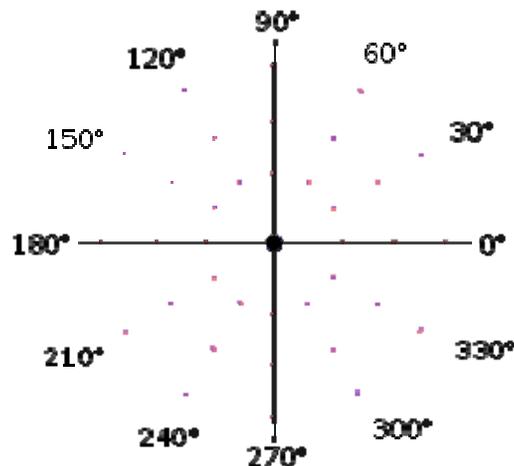
$$y = r \sin \theta, \quad (2)$$

where  $r$  is the radial distance from the origin, and  $\theta$  is the counterclockwise angle from the  $x$ -axis. In terms of  $x$  and  $y$ ,

$$r = \sqrt{x^2 + y^2} \quad (3)$$

$$\theta = \tan^{-1} \left( \frac{y}{x} \right) \quad (4)$$

(Here,  $\tan^{-1} (y/x)$  should be interpreted as the two-argument inverse tangent which takes the signs of  $x$  and  $y$  into account to determine in which quadrant  $\theta$  lies.)



Sheet based on:

[http://en.wikipedia.org/wiki/Polar\\_coordinate\\_system](http://en.wikipedia.org/wiki/Polar_coordinate_system)

<http://mathworld.wolfram.com/PolarCoordinates.html>

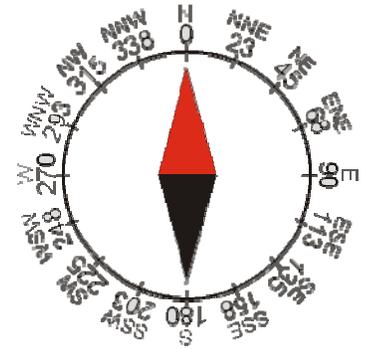
## Cardinal Coordinate System

In mathematics, the cardinal coordinate system is a two-dimensional coordinate system in which each point on a plane is determined by a distance from a fixed point and an angle from a fixed direction.

The fixed point (analogous to the origin of a Cartesian system) is called the pole, and the ray from the pole with the fixed direction is the cardinal axis. The distance from the pole is called the radial coordinate or radius, and the angle is the angular coordinate, polar angle, or azimuth.

The way we state cardinal coordinates often confuses people. If we say something is moving at  $30^\circ$  east or north. That means find north and then go  $30^\circ$  east. This same point can also be read  $60^\circ$  north of east or simply  $60^\circ$  northeast.

Another example would be  $20^\circ$  south of west, which means find west and then go 20 degrees south of that value. Another way to name this is  $70^\circ$  west of south.



The cardinal coordinates  $r$  (the radial coordinate) and  $\theta$  (the angular coordinate, often called the cardinal angle) are defined in terms of Polar coordinates by adding or subtracting their values from the reference angle.

For example

$0^\circ$  c.c. is the same as  $90^\circ$  p.c.

$90^\circ$  c.c. is the same as  $0^\circ$  p.c.

$180^\circ$  c.c. is the same as  $270^\circ$  p.c.

$270^\circ$  c.c. is the same as  $180^\circ$  p.c.

All other angles can be converted to p.c. by referencing the cardinal directions.

To convert these coordinates to the Cartesian system, it is recommended, they be converted to the polar directions first.

