Chapter 13 Problems

1, **2**, **3** = straightforward, intermediate, challenging

Section 13.1 Hooke's Law

1. A 0.40-kg object is attached to a spring with a spring constant 160 N/m so that the object is allowed to move on a horizontal frictionless surface. The object is released from rest when the spring is compressed 0.15 m. Find (a) the force on the object and (b) its acceleration at this instant.

2. A load of 50 N attached to a spring hanging vertically stretches the spring 5.0 cm. The spring is now placed horizontally on a table and stretched 11 cm. (a) What force is required to stretch the spring by this amount? (b) Plot a graph of force (on the y axis) versus spring displacement from the equilibrium position along the x axis.

3. A ball dropped from a height of 4.00 m makes a perfectly elastic collision with the ground. Assuming no mechanical energy is lost due to air resistance, (a) show that the motion is periodic and (b) determine the period of the motion. (c) Is the motion simple harmonic? Explain.

4. The four springs of the device shown in Figure P13.4 each have a spring constant of 65.0 N/m. The hands holding the unstretched device are 43.5 cm apart. If the person cannot move his hands more than 80.0 cm apart, what is the maximum lateral force each hand can exert?

FIGURE P13.4

5. The springs 1 and 2 in Figure P13.5 have spring constants of 40.0 N/cm and 25.0 N/cm, respectively. The object A remains at rest, and both springs are stretched equally. Determine the stretch.
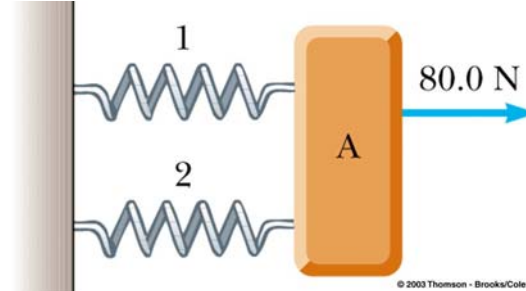

FIGURE P13.5

6. (a) The spring in Figure P13.6 has a force constant of 5.60 x 10^4 N/m. If the tension in either side of the rope is 210 N, how far is the spring stretched? (b) Suppose the rope is replaced by two springs which make the same V shape as the rope. If the stretch of each of these springs is twice that of the lower spring, what must be the spring constant of these springs?
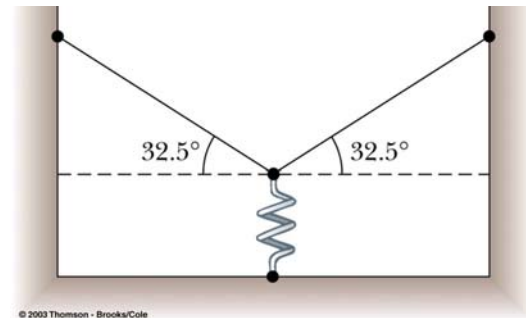

FIGURE P13.6

Section 13.2 Elastic Potential Energy

7. A slingshot consists of a light leather cup, containing a stone, that is pulled back against two parallel rubber bands. It takes a force of 15 N to stretch either one of these bands 1.0 cm. (a) What is the potential energy stored in the two bands together when a 50-g stone is placed in the cup and pulled back 0.20 m from the equilibrium position? (b) With what speed does the stone leave the slingshot?

8. An archer pulls her bow string back 0.400 m by exerting a force that increases uniformly from zero to 230 N. (a) What is the equivalent spring constant of the bow? (b) How much work is done in pulling the bow?

9. A child's toy consists of a piece of plastic attached to a spring (Fig. P13.9). The spring is compressed against the floor a distance of 2.00 cm, and the toy is released. If the toy has a mass of 100 g and rises to a maximum height of 60.0 cm, estimate the force constant of the spring.

FIGURE P13.9

10. An automobile having a mass of 1 000 kg is driven into a brick wall in a safety test. The bumper behaves like a spring of constant 5.00×10^6 N/m and compresses 3.16 cm as the car is brought to rest. What was the speed of the car before impact, assuming no energy is lost during impact with the wall?

11. A simple harmonic oscillator has a total energy of E. (a) Determine the kinetic and potential energies when the displacement is one half the amplitude. (b) For what value of the displacement does the kinetic energy equal the potential energy?

12. A 1.50-kg block at rest on a tabletop is attached to a horizontal spring having constant 19.6 N/m as in Figure P13.12. The spring is initially unstretched. A constant 20.0-N horizontal force is applied to the object causing the spring to stretch. (a) Determine the speed of the block after it has moved 0.300 m from equilibrium if the surface between the block and tabletop is frictionless. (b) Answer part (a) if the coefficient of kinetic friction between block and tabletop is 0.200.
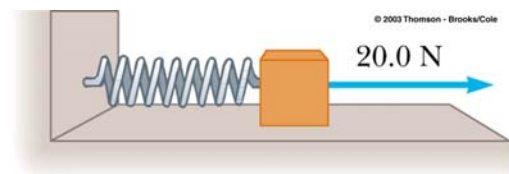
20.0 N

FIGURE P13.12

13. A 10.0-g bullet is fired into and embeds itself in a 2.00-kg block attached to a spring with a spring constant of 19.6 N/m and whose mass is negligible. How far is the spring compressed if the bullet has a speed of 300 m/s just before it strikes the block, and the block slides on a frictionless surface? (*Note:* You must use conservation of momentum in this problem. Why?)

14. A 1.5-kg block is attached to a spring with a spring constant of 2 000 N/m. The spring is then stretched a distance of 0.30 cm and the block is released from rest. (a) Calculate the speed of the block as it passes through the equilibrium position if no friction is present. (b) Calculate the speed of the block as it passes through the equilibrium position if a constant frictional force of 2.0 N retards its motion. (c) What would be the strength of the frictional force if the block reached the equilibrium position the first time with zero velocity?

Section 13.3 Velocity as a Function of Position

15. A 0.40-kg object connected to a light spring with a spring constant of 19.6 N/m oscillates on a frictionless horizontal surface. If the spring is compressed 4.0 cm and released from rest, determine (a) the maximum speed of the object, (b) the speed of the object when the spring is compressed 1.5 cm, and (c) the speed of the object when the spring is stretched 1.5 cm. (d) For what value of x does the speed equal one half of the maximum speed?

16. An object-spring system oscillates with an amplitude of 3.5 cm. If the spring constant is 250 N/m and the object has a mass of 0.50 kg, determine (a) the mechanical energy of the system, (b) the maximum speed of the object, and (c) the maximum acceleration.

17. At an outdoor market, a bunch of bananas is set into oscillatory motion with an amplitude of 20.0 cm on a spring with a spring constant of 16.0 N/m. It is observed that the maximum speed of the bunch of bananas is 40.0 cm/s. What is the weight of the bananas in newtons?

18. A 50.0-g object is attached to a horizontal spring with a spring constant of 10.0 N/m and released from rest with an amplitude of 25.0 cm. What is the velocity of the object when it is halfway to the equilibrium position if the surface is frictionless?

Section 13.4 Comparing Simple Harmonic Motion with Uniform Circular Motion

19. While riding behind a car traveling at 3.00 m/s, you notice that one of the car's tires has a small hemispherical bump on its rim, as in Figure P13.19. (a) Explain why the

bump, from your viewpoint behind the car, executes simple harmonic motion. (b) If the radius of the car's tires is 0.30 m, what is the bump's period of oscillation?
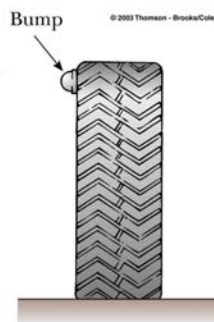
FIGURE P13.19

20. An object moves uniformly around a circular path of radius 20.0 cm, making one complete revolution every 2.00 s. What are the (a) translational speed of the object, (b) the frequency of motion in hertz, and (c) the angular speed of the object?

21. Consider the simplified single-piston engine in Figure 13.21. If the wheel rotates at a constant angular speed ω, explain why the piston rod oscillates in simple harmonic motion.
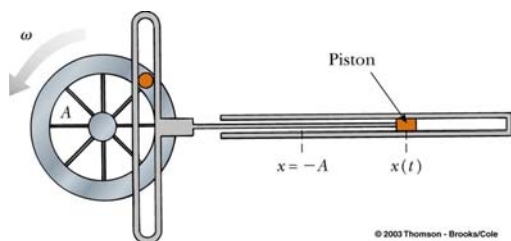
FIGURE P13.21

22. A 200-g object is attached to a spring and executes simple harmonic motion with a period of 0.250 s. If the total energy of the system is 2.00 J, find (a) the force constant of the spring and (b) the amplitude of the motion.

23. A spring stretches 3.9 cm when a 10-g object is hung from it. The object is replaced with a block of mass of 25 g which oscillates in simple harmonic motion. Calculate the period of motion.

24. When four people with a combined mass of 320 kg sit down in a car, they find that the car drops 0.80 cm lower on its springs. Then they get out of the car and bounce it up and down. What is the

frequency of the car's vibration if its mass (empty) is 2.0 × 10^3 kg?

25. A 5.50-g object is suspended from a cylindrical sample of collagen 3.50 cm long and 2.00 mm in diameter. If the object vibrates up and down with a frequency of 36.0 Hz, what is the Young's modulus of the collagen?

Section 13.5 Position, Velocity, and Acceleration as a Function of Time

26. The motion of an object is described by the equation

$$x = (0.30 \text{ m}) \cos(\pi t/3)$$

Find (a) the position of the object at $t = 0$ and $t = 0.60$ s, (b) the amplitude of the motion, (c) the frequency of the motion, and (d) the period of the motion.

27. A 2.00-kg object on a frictionless horizontal track is attached to the end of a horizontal spring whose force constant is 5.00 N/m. The object is displaced 3.00 m to the right from its equilibrium position and then released, which initiates simple harmonic motion. (a) What is the force (magnitude and direction) acting on the object 3.50 s after it is released? (b) How many times does the object oscillate in 3.50 s?

28. A spring of negligible mass stretches 3.00 cm from its relaxed length when a force of 7.50 N is applied. A 0.500-kg particle rests on a frictionless horizontal surface and is attached to the free end of the spring. The particle is pulled horizontally so that it stretches the spring 5.00 cm and is then released from rest at $t = 0$. (a) What is the force constant of the spring? (b) What are the angular frequency ω, the frequency, and the period of the motion? (c) What is the total energy of the system? (d) What is the amplitude of the motion? (e) What are the maximum velocity and the maximum acceleration of the particle? (f) Determine the displacement x of the particle from the equilibrium position at $t = 0.500$ s.

29. Given that $x = A \cos \omega t$ is a sinusoidal function of time, show that v (velocity) and a (acceleration) are also sinusoidal functions of time. (*Hint:* Use Equations 13.6 and 13.2.)

Section 13.6 Motion of a Pendulum

30. A man enters a tall tower, needing to know its height. He notes that a long pendulum extends from the ceiling almost to the floor and that its period is 15.5 s. (a) How tall is the tower? (b) If this pendulum is taken to the Moon, where the free-fall acceleration is 1.67 m/s^2, what is the period there?

31. A "seconds" pendulum is one that moves through its equilibrium position once each second. (The period of the pendulum is 2.000 s.) The length of a seconds pendulum is 0.992 7 m at Tokyo and 0.994 2 m at Cambridge, England. What is the ratio of the free-fall accelerations at these two locations?

32. An aluminum clock pendulum having a period of 1.00 s keeps perfect time at 20.0°C. (a) When placed in a room at a temperature of –5.0°C, will it gain time or lose time? (b) How much time will it gain or lose every hour? (*Hint:* See Chapter 10.)

33. A pendulum clock that works perfectly on Earth is taken to the Moon. (a) Does it run fast or slow there? (b) If the clock is started at 12:00 midnight, what will it read after one Earth-day (24.0 h)? Assume that the free-fall acceleration on the Moon is 1.63 m/s^2.

34. A simple pendulum is 5.00 m long. (a) What is the period of simple harmonic motion for this pendulum if it is located in an elevator accelerating upward at 5.00 m/s$^{2?}$ (b) What is its period if the elevator is accelerating downward at 5.00 m/s^2? (c) What is the period of simple harmonic motion for this pendulum if it is placed in a truck that is accelerating horizontally at 5.00 m/s^2.

35. The free-fall acceleration on Mars is 3.7 m/s^2. (a) What length pendulum has a period of 1 s on Earth? What length pendulum would have a 1-s period on Mars? (b) An object is suspended from a spring with spring constant 10 N/m. Find the mass suspended from this spring that would result in a period of 1 s on Earth and on Mars.

36. Two playground swings start out together. After ten complete oscillations the swings are out of step by a half-cycle. Find the percentage difference in the length of the swings.

Section 13.10 Frequency, Amplitude, and Wavelength

37. A wave traveling in the positive x direction has a frequency of 25.0 Hz as in Figure P13.37. Find the (a) amplitude, (b) wavelength, (c) period, and (d) speed of the wave.
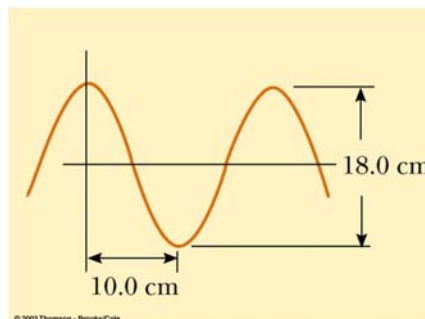
FIGURE P13.37

38. A bat can detect small objects such as an insect whose size is approximately equal to one wavelength of the sound the bat makes. If bats emit a chirp at a frequency of 60.0 kHz, and if the speed of sound in air is 340 m/s, what is the smallest insect a bat can detect?

39. If the frequency of oscillation of the wave emitted by an FM radio station is 88.0 MHz, determine the wave's (a) period of vibration and (b) wavelength. (Radio waves travel at the speed of light, 3.00 x 10^8 m/s.)

40. A piano emits sound waves with frequencies that range from a low of about 28 Hz to a high of about 4 200 Hz. Find the range of wavelengths spanned by this instrument. The speed of sound in air is approximately 343 m/s.

41. A harmonic wave is traveling along a rope. It is observed that the oscillator that generates the wave completes 40.0 vibrations in 30.0 s. Also, a given maximum travels 425 cm along the rope in 10.0 s. What is the wavelength?

42. Ocean waves are traveling to the east at 4.0 m/s with a distance of 20 m between crests. With what frequency do the waves hit the front of a boat (a) when the boat is at anchor and (b) when the boat is moving westward at 1.0 m/s?

Section 13.11 The Speed of Waves on Strings

43. A phone cord is 4.00 m long. The cord has a mass of 0.200 kg. A transverse wave pulse is produced by plucking one end of the taut cord. The pulse makes four trips down and back along the cord in 0.800 s. What is the tension in the cord?

44. A circus performer stretches a tightrope between two towers. He strikes one end of the rope and sends a wave along it toward the other tower. He notes that it takes the wave 0.800 s to reach the opposite tower, 20.0 m away. If a 1-m length of the rope has a mass of 0.350 kg, find the tension in the tightrope.

45. Transverse waves with a speed of 50.0 m/s are to be produced on a stretched string. A 5.00-m length of string with a total mass of 0.060 0 kg is used. (a) What is the required tension in the string? (b) Calculate the wave speed in the string if the tension is 8.00 N.

46. An astronaut on the Moon wishes to measure the local value of g by timing pulses traveling down a wire that has a large object suspended from it. Assume a wire of mass 4.00 g is 1.60 m long and has a 3.00-kg object suspended from it. A pulse requires 36.1 ms to traverse the length of the wire. Calculate g_{Moon} from these data. (You may neglect the mass of the wire when calculating the tension in it.)

47. A simple pendulum consists of a ball of mass 5.00 kg hanging from a uniform string of mass 0.060 0 kg and length L. If the period of oscillation for the pendulum is 2.00 s, determine the speed of a transverse wave in the string when the pendulum hangs vertically.

48. A string is 50.0 cm long and has a mass of 3.00 g. A wave travels at 5.00 m/s along this string. A second string has the same length but half the mass of the first. If the two strings are under the same tension, what is the speed of a wave along the second string?

49. Tension is maintained in a string as in Figure P13.49. The observed wave speed is 24 m/s when the suspended mass is 3.0 kg. (a) What is the mass per unit length of the string? (b) What is the wave speed when the suspended mass is 2.0 kg?
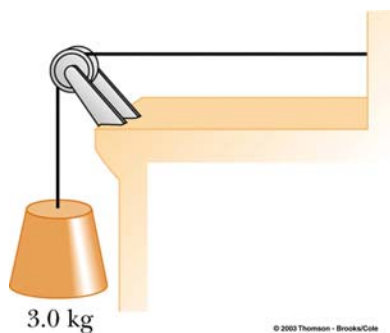

3.0 kg

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FIGURE P13.49

50. The elastic limit of a piece of steel wire is 2.70 x 10^9 Pa. What is the maximum speed at which transverse wave pulses can propagate along this

wire without exceeding this stress? (The density of steel is 7.86 x 10^3 kg/m^3).

51. Transverse waves travel at 20.0 m/s on a string that is under a tension of 6.00 N. What tension is required for a wave speed of 30.0 m/s in the same string?

Section 13.12 Interference of Waves Section

13.13 Reflection of Waves

52. A series of pulses of amplitude 0.15 m are sent down a string that is attached to a post at one end. The pulses are reflected at the post and travel back along the string without loss of amplitude. What is the amplitude at a point on the string where two pulses are crossing, (a) if the string is rigidly attached to the post? (b) if the end at which reflection occurs is free to slide up and down?

53. A wave of amplitude 0.30 m interferes with a second wave of amplitude 0.20 m traveling in the same direction. What are the (a) largest and (b) smallest resultant amplitudes that can occur, and under what conditions will these maxima and minima occur?

Additional Problems

54. The position of a 0.30-kg object attached to a spring is described by

$$x = (0.25 \text{ m})\cos(0.4 \pi t)$$

Find (a) the amplitude of the motion, (b) the spring constant, (c) the position at $t = 0.30$ s, and (d) the object's speed at $t = 0.30$ s.

55. A large block P executes horizontal simple harmonic motion as it slides across a frictionless surface with a frequency $f = 1.50$ Hz. Block B rests on it, as shown in Figure P13.55, and the coefficient of static friction between the two is $\mu_s = 0.600$. What maximum amplitude of oscillation can the system have if block B is not to slip?
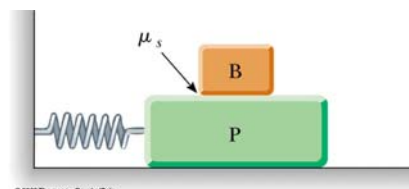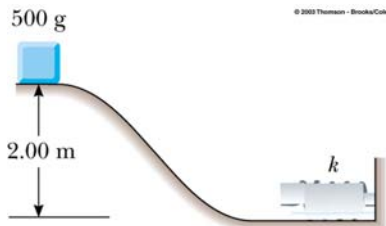

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FIGURE P13.55

56. A 500-g block is released from rest and slides down a frictionless track that begins 2.00 m above the horizontal, as shown in Figure P13.56. At the bottom of the track, where

the surface is horizontal, the block strikes and sticks to a light spring with a spring constant of 20.0 N/m. Find the maximum distance the spring is compressed.
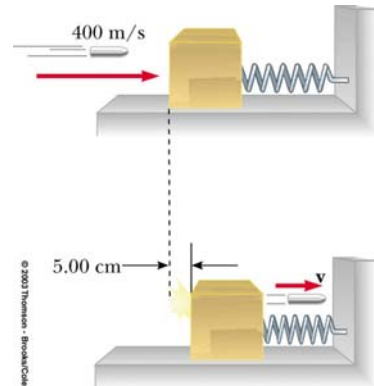

FIGURE P13.56

57. A 3.00-kg object is fastened to a light spring with the intervening cord passing over a pulley (Fig. P13.57). The pulley is frictionless, and its inertia may be neglected. The object is released from rest when the spring is unstretched. If the object drops 10.0 cm before stopping, find (a) the spring constant of the spring and (b) the speed of the object when it is 5.00 cm below its starting point.
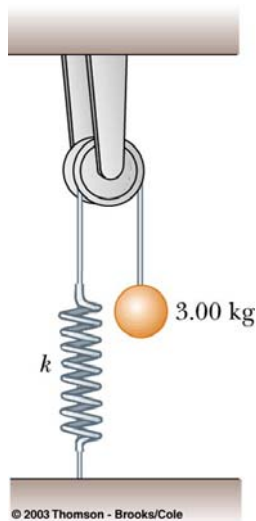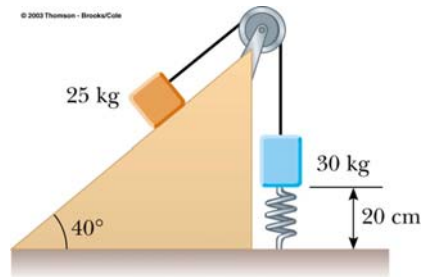

FIGURE P13.57

58. A 5.00-g bullet moving with an initial speed of 400 m/s is fired into and passes through a 1.00-kg block, as in Figure P13.58. The block, initially at rest on a frictionless horizontal surface, is connected to a spring with a spring constant of 900 N/m. If the block moves 5.00 cm to the right after impact, find (a) the speed at which the bullet emerges from the block and (b) the mechanical energy lost in the collision.

FIGURE P13.58

59. A 25-kg block is connected to a 30-kg block by a light string that passes over a frictionless pulley. The 30-kg block is connected to a light spring of force constant 200 N/m, as in Figure P13.59. The spring is unstretched when the system is as shown in the figure, and the incline is smooth. The 25-kg block is pulled 20 cm down the incline (so that the 30-kg block is 40 cm above the floor) and is released from rest. Find the speed of each block when the 30-kg block is 20 cm above the floor (that is, when the spring is unstretched).

FIGURE P13.59

60. A 30.0-m steel wire and a 20.0-m copper wire, both with 1.00-mm diameters, are connected end to end and stretched to a tension of 150 N. How long does it take a transverse wave to travel the entire length of the two wires?

61. A 2.00-kg block hangs without vibrating at the end of a spring ($k = 500$ N/m) that is attached to the ceiling of an elevator car. The car is rising with an upward acceleration of $g/3$ when the acceleration suddenly ceases (at $t = 0$). (a) What is the angular frequency of oscillation of the block after the acceleration ceases? (b) By what amount is the spring stretched during the time that the elevator car is accelerating? This distance will be the amplitude of the ensuing oscillation of the block.

62. An object of mass m is connected to two rubber bands of length L, each under tension F, as in Figure P13.62. The object is displaced vertically by a small distance y. Assuming the tension does not change, show that (a) the restoring force

is $-(2F/L)y$ and (b) the system exhibits simple harmonic motion with an angular frequency
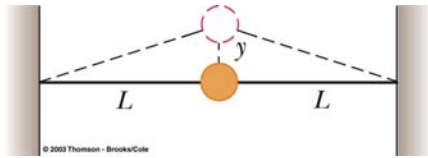
$$\omega = \sqrt{2F/mL}\,.$$

FIGURE P13.62

63. A light balloon filled with helium of density 0.180 kg/m³ is tied to a light string of length $L =$ 3.00 m. The string is tied to the ground, forming an "inverted" simple pendulum (Fig. P13.63a). If the balloon is displaced slightly from equilibrium, as in Figure P13.63b, show that the motion is simple harmonic, and determine the period of the motion. Take the density of air to be 1.29 kg/m³. (*Hint:* Use an analogy with the simple pendulum discussed in the text, and see Chapter 9.)

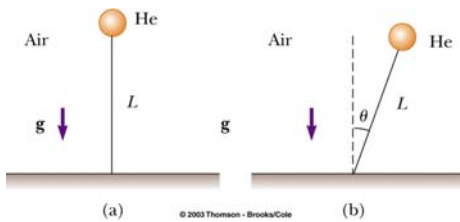**FIGURE P13.63**

64. A light string of mass 10.0 g and length $L =$ 3.00 m has its ends tied to two walls that are separated by the distance $D =$ 2.00 m. Two objects, each of mass $m =$ 2.00 kg, are suspended from the string as in Figure P13.64. If a wave pulse is sent from point A, how long does it take to travel to point B?
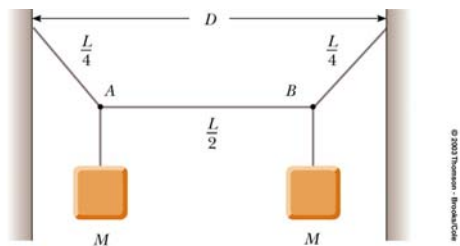

FIGURE P13.64

65. Assume that a hole is drilled through the center of Earth. It can be shown that an object of mass m at a distance r from Earth's center is pulled toward the center of the Earth only by the material in the shaded portion of Figure P13.65. Assume Earth has

a uniform density ρ. Write down Newton's law of gravitation for an object at distance r from the center of Earth, and show that the force on it is of Hooke's law form, $F = -kr$, with an effective force constant of $k = 4/3\pi\rho Gm$, where G is the gravitational constant.
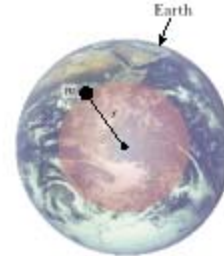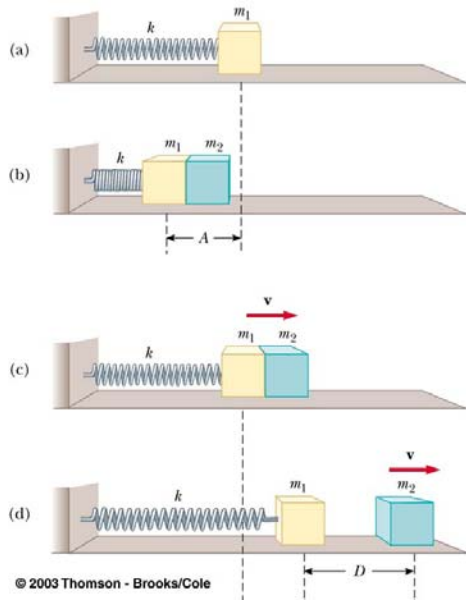

FIGURE P13.65

66. A 60.0-kg firefighter slides down a pole while a constant frictional force of 300 N retards his motion. A horizontal 20.0-kg platform is supported by a spring at the bottom of the pole to cushion the fall. The firefighter starts from rest 5.00 m above the platform, and the spring constant is 2 500 N/m. Find (a) the firefighter's speed just before he collides with the platform and (b) the maximum distance the spring is compressed. Assume that the frictional force acts during the entire motion. (*Hint:* The collision between the firefighter and the platform is perfectly inelastic.)

67. An object of mass $m_1 =$ 9.0 kg is in equilibrium while connected to a light spring of constant $k =$ 100 N/m that is fastened to a wall as in Figure P13.67a. A second object of mass $m_2 =$ 7.0 kg is slowly pushed up against m_1, compressing the spring by the amount $A =$ 0.20 m, as shown in Figure P13.67b. The system is then released, causing both objects to start moving to the right on the frictionless surface. (a) When m_1 reaches the equilibrium point, m_2 loses contact with m_1 (Fig. P13.67c) and moves to the right with velocity v. Determine the magnitude of v. (b) How far apart are the objects when the spring is fully stretched for the first time (Fig. P13.67d)? (*Hint:* First determine the period of oscillation and the amplitude of the m_1 -spring system after m_2 loses contact with m_1.)

(a)

(b)

(c)

(d)

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FIGURE P13.67

68. An 8.00-kg block travels on a rough horizontal surface and collides with a spring. The speed of the block just before the collision is 4.00 m/s. As it rebounds to the left with the spring uncompressed, the block travels at 3.00 m/s. If the coefficient of kinetic friction between the block and the surface is 0.400, determine (a) the loss in mechanical energy due to friction while the block is in contact with the spring and (b) the maximum distance the spring is compressed.