Types of Coordinate Systems

Based on http://cnx.org/content/m13600/latest/

Coordinate system is a system of measurement of distance and direction with respect to rigid bodies. Structurally, it comprises of coordinates and a reference point, usually the origin of the coordinate system. The coordinates primarily serve the purpose of reference for the direction of motion, while origin serves the purpose of reference for the magnitude of motion.

Measurements of magnitude and direction allow us to locate a position of a point in terms of measurable quantities like linear distances or angles or their combinations. With these measurements, it is possible to locate a point in the spatial extent of the coordinate system. The point may lie anywhere in the spatial (volumetric) extent defined by the rectangular axes as shown in the figure. (Note : The point, in the figure, is shown as small sphere for visual emphasis only)

A point in the coordinate system

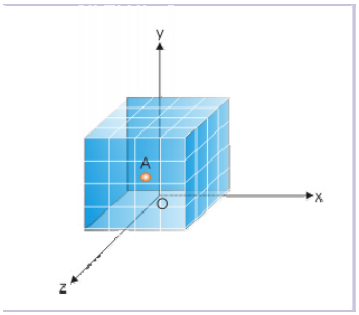

Figure 1

A distance in the coordinate system is measured with a standard rigid linear length like that of a "meter" or a "foot". A distance of 5 meters, for example, is 5 times the length of the standard length of a meter. On the other hand, an angle is defined as a ratio of lengths and is dimensional-less. Hence, measurement of direction is indirectly equivalent to the measurement of distances only.

The coordinate system represents the system of rigid body like earth, which is embodied by an observer, making measurements. Since measurements are implemented by the observer, they (the measurements in the coordinate system) represent distance and direction as seen by the observer. It is, therefore, clearly implied that measurements in the coordinates system are specific to the state of motion of the coordinate system.

In a plane language, we can say that the description of motion is specific to a system of rigid bodies, which involves measurement of distance and direction. The measurements are done, using standards of length, by an observer, who is at rest with the system of rigid bodies. The observer makes use of a coordinate system attached to the system of rigid bodies and uses the same as reference to make measurements.

It is apparent that the terms "system of rigid bodies", "observer" and "coordinate system" etc. are similar in meaning; all of which conveys a system of reference for carrying out measurements to describe motion. We sum up the discussion thus far as :

1. Measurements of distance, direction and location in a coordinate system are specific to the system of rigid bodies, which serve as reference for both magnitude and direction.

2. Like point, distance and other aspects of motion, the concept of space is specific to the reference represented by coordinate system. It is, therefore, suggested that use of word "space" independent of coordinate system should be avoided and if used it must be kept in mind that it represents volumetric extent of a specific coordinate system. The concept of space, if used without caution, leads to an inaccurate understanding of the laws of nature.

3. Once the meanings of terms are clear, "the system of reference" or "frame of reference" or "rigid body system" or "observer" or "coordinate system" may be used interchangeably to denote an unique system for determination of motional quantities and the representation of a motion.

Coordinate system types

Coordinate system types determine position of a point with measurements of distance or angle or combination of them. A spatial point requires three measurements in each of these coordinate types. It must, however, be noted that the descriptions of a point in any of these systems are equivalent. Different coordinate types are mere convenience of appropriateness for a given situation. Three major coordinate systems used in the study of physics are :

- Rectangular (Cartesian)
- Spherical
- Cylindrical

Rectangular (Cartesian) coordinate system is the most convenient as it is easy to visualize and associate with our perception of motion in daily life. Spherical and cylindrical systems are specifically designed to describe motions, which follow spherical or cylindrical curvatures.

Rectangular (Cartesian) coordinate system

The measurements of distances along three mutually perpendicular directions, designated as x,y and z, completely define a point A (x,y,z).

A point in rectangular coordinate system

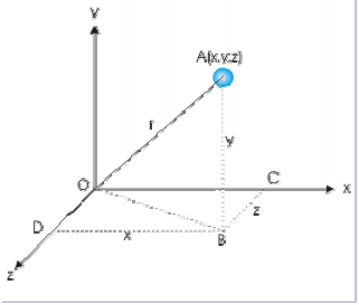

Figure 2: A point is specified with three coordinate values

Spherical coordinate system

A three dimensional point "A" in spherical coordinate system is considered to be located on a sphere of a radius "r". The point lies on a particular cross section (or plane) containing origin of the coordinate system. This cross section makes an angle "θ" from the "zx" plane (also known as longitude angle). Once the plane is identified, the angle, ϕ, that the line joining origin O to the point A, makes with "z" axis, uniquely defines the point A (r, θ, ϕ).

Spherical coordinate system

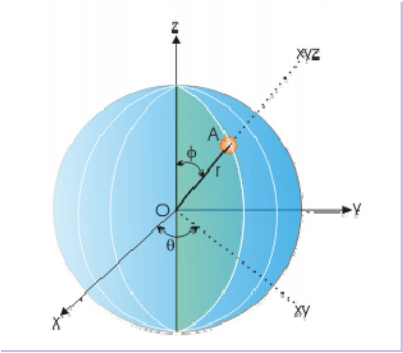

Figure 5: A point is specified with three coordinate values

Cylindrical coordinate system

A three dimensional point "A" in cylindrical coordinate system is considered to be located on a cylinder of a radius "r". The point lies on a particular cross section (or plane) containing origin of the coordinate system. This cross section makes an angle "θ" from the "zx" plane. Once the plane is identified, the height, z, parallel to vertical axis "z" uniquely defines the point A(r, θ, z)

Cylindrical coordinate system

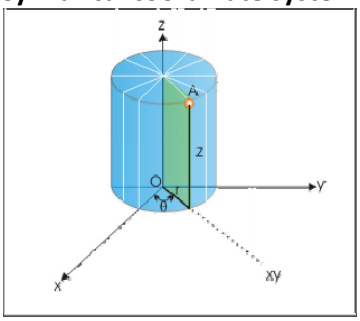

Figure 7: A point is specified with three coordinate values Cylindrical coordinate system