LESSON 6.2 DIMENSIONAL ANALYSIS

In Lesson 4.6, you used the distance formula $D = rt$. D is the distance traveled while moving at a rate (or speed) r during a period of time t. If you solve the distance formula for r, rate is a ratio.

$$r = \frac{D}{t}$$

In general, a **rate** is a ratio that compares two unlike quantities. For example, if a truck travels 100 miles in 2 hours, the rate is

$$r = \frac{D}{t} = \frac{100 \text{ miles}}{2 \text{ hours}}$$

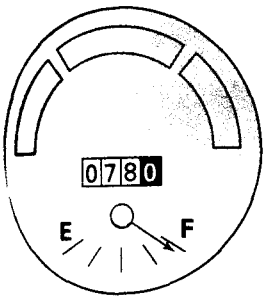
A **unit rate** is a comparison to one unit. To write a unit rate, find an equivalent ratio with 1 as the denominator. You can use equal ratios to find unit rates. For the example above,

$$\frac{\text{Miles} \rightarrow}{\text{Hours} \rightarrow} \frac{100}{2} = \frac{50}{1}$$

The unit rate is 50 miles for each hour traveled. You can write 50 miles per hour *or* 50 mi/hr *or* 50 $\frac{\text{mi}}{\text{hr}}$. The word *per* is often used in place of *for each*. You can abbreviate *per* with the / symbol.

ACTIVITY **Finding Unit Rates**

Write unit rates for each of the following. The first one is done as an example.



1 Sale price: 5 cans for $2

$$\frac{5 \text{ cans}}{2 \text{ dollars}} = \frac{2.5 \text{ cans}}{1 \text{ dollar}} = 2.5 \text{ cans/dollar}$$

2 Gas mileage: 78 miles using 3 gallons of gas

$$\frac{78 \text{ miles}}{3 \text{ gallons}} = \frac{? \text{ miles}}{1 \text{ gallon}} = \underline{?} \text{ mi/gal}$$

3 Starting wage: $326 for 40 hours

$$\frac{326 \text{ dollars}}{40 \text{ hours}} = \frac{? \text{ dollars}}{1 \text{ hour}} = \$ \underline{?} /\text{hr}$$

4 Weight of oranges: 5 oranges weigh 0.75 kilograms

$$\frac{5 \text{ oranges}}{0.75 \text{ kg}} = \frac{?}{?} = \underline{?} \text{ oranges/kg}$$

Ongoing Assessment

In the 1996 Summer Olympic Games, a world record of 19.32 seconds was set in the 200-meter dash. A world record of 9.84 seconds was set in the 100-meter dash. Did the runner of the 200-meter dash or the runner of the 100-meter dash run the fastest average speed?

When solving problems, you often need to change or convert the units of measurement of a quantity. To convert units of measurement, multiply by a *conversion factor*.

A **conversion factor** is a ratio in which the numerator equals the denominator, but in different units. This ratio is equal to 1.

For example, a length of 12 inches equals a length of 1 foot. Thus,

$$12 \text{ in.} = 1 \text{ ft} \qquad \text{and} \qquad \frac{12 \text{ in.}}{1 \text{ ft}} = 1$$

Example 1 Using Conversion Factors

Use the following conversion factors.

$$\frac{5280 \text{ ft}}{1 \text{ mi}} \qquad \frac{60 \text{ min}}{1 \text{ hr}} \qquad \frac{24 \text{ hr}}{1 \text{ day}} \qquad \frac{1 \text{ week}}{7 \text{ day}}$$

a. Convert 1.75 miles to feet.

b. Convert 6800 feet to miles.

c. Convert 1 week to minutes.

Solution

a. $1.75 \text{ mi} = \frac{1.75 \text{ mi}}{1} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} = 9240 \text{ ft}$

b. Since $\frac{5280 \text{ ft}}{1 \text{ mi}} = 1$, its reciprocal $\frac{1 \text{ mi}}{5280 \text{ ft}} = 1$.

$$6800 = \frac{6800 \text{ ft}}{1} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} = \text{about } 1.288 \text{ mi}$$

c. $1 \text{ week} = \frac{1 \text{ week}}{1} \cdot \frac{7 \text{ day}}{1 \text{ week}} \cdot \frac{24 \text{ hr}}{1 \text{ day}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} = 10{,}080 \text{ min}$

Ongoing Assessment

Use the conversion factors in Example 1.

a. Convert 18,480 feet to miles.

b. Convert 100,000 minutes to weeks.

Critical Thinking Use the following equalities to write ratios that can be used as conversion factors. Write two ratios from each equality. Why is each ratio equal to 1?

2.54 cm = 1 in. 1 kg = 2.205 lb 1.467 ft/sec = 1 mi/hr

EXAMPLE 2 Converting Units

Ichiro has taken a summer job that pays $7.50 per hour. At this rate, how much money will Ichiro earn working 8 hours per day, 5 days per week, for 6 weeks?

SOLUTION

Since Ichiro works 6 weeks, find the unit rate in dollars per week.

$$\frac{7.50 \text{ dollars}}{1 \text{ hr}} \cdot \frac{8 \text{ hr}}{1 \text{ day}} \cdot \frac{5 \text{ days}}{1 \text{ week}} = \frac{300 \text{ dollars}}{1 \text{ week}}$$

$$\text{Earnings} = \text{Rate} \cdot \text{Time}$$

$$= \frac{300 \text{ dollars}}{1 \text{ week}} \cdot 6 \text{ weeks}$$

$$= 1800 \text{ dollars}$$

Ichiro will earn $1800 in 6 weeks.

LESSON ASSESSMENT

Think and Discuss

1. How do you write a ratio as a unit rate?
2. Explain why you can use the reciprocal of any conversion factor as a conversion factor.
3. How do you know whether to use a given conversion factor or its reciprocal?

Practice and Apply

Write a unit rate for each of the following.

4. 14 boxes of files cost $33.60.

5. 54 ounces has a volume of 20 cubic centimeters.

6. 10 gallons is equivalent to 37.7 liters.

7. Walking 29.3 feet in 20 seconds.