

## Archimedes' Principle (Force Sensor)

Buoyancy is the ability of a fluid to sustain a body floating in it or to diminish the apparent weight of a body submerged in it. Archimedes' principle states that this apparent reduction in weight is equal to the weight of the fluid displaced. It is the purpose of this experiment to study Archimedes' principle and its application to the determination of density and specific gravity. In particular, the specific gravities of a solid heavier than water, a solid lighter than water, and a liquid other than water will be measured.

### THEORY

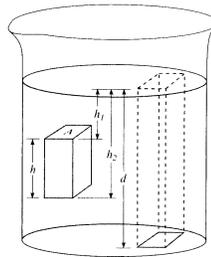


Figure 1 A beaker of fluid with a submerged rectangular object

The density of a body is defined as its mass per unit volume. It is usually expressed in grams per cubic centimeter.\* The specific gravity of a body is the ratio of its density to the density of water at the same temperature. Since for a given location the weight of a body is taken as a measure of its mass, the specific gravity may be taken as the ratio of the weight of a given volume of a substance to the weight of an equal volume of water. Because the mass of 1 cm<sup>3</sup> of water at 4°C is 1 g, the specific gravity of a body at this temperature is also numerically equal to its density in grams per cubic centimeter.

If a body is totally immersed in a fluid, the volume of fluid displaced must be equal to the volume of the body, because if the body weren't there, its volume would be occupied by the fluid. Moreover, the hydrostatic pressure on the bottom of the body is greater than that on the top because hydrostatic pressure rises as depth increases. The situation is illustrated in Fig. 1, which shows a beaker of fluid with an object (which we have made rectangular in the interest of simplicity) submerged in it. Let us begin by imagining an area of 1 cm<sup>2</sup> on the bottom of the beaker as shown at the right in Fig. 1. The dotted lines indicate that we may think of this 1-cm<sup>2</sup> patch as supporting a column of fluid 1 cm<sup>2</sup> in cross section extending up to the surface a distance  $d$  above it, the depth of fluid being  $d$  cm. Thus a volume  $1 \times d$  cm<sup>3</sup> and hence a weight of  $\rho g d$  dynes of the fluid sitting on that 1 cm<sup>2</sup> of the bottom. Here  $\rho$  is the fluid's density and  $g$  is the acceleration of gravity in centimeters per second squared. Hence the pressure sustained by the bottom of the beaker is  $\rho g d$  dynes weighing on every square centimeter, or  $\rho g d$  dynes/cm<sup>2</sup>. Note that this is the gauge pressure. The *absolute* pressure must include the pressure due to the atmosphere on the fluid surface—we could imagine the dotted lines in Fig. 1 going on up to the top of the atmosphere and indicating a column of air 1 cm<sup>2</sup> in cross section sitting on top of the column of liquid. Since atmospheric pressure does not affect the present experiment we will not consider it further.

Turning now to the immersed solid, we note that it has height  $h$ , its bottom end being at depth  $h_2$  and its top end at depth  $h_1$  where  $h_2 - h_1 = h$ . The considerations of the last paragraph show that the pressure at any depth  $d$  in a fluid of density  $\rho$  is  $\rho g d$ ; hence the pressure on the bottom of the body is  $\rho g h_2$  and the force exerted on the bottom, which will be an upward force, is  $\rho g h_2 A$ , where  $A$  is the body's cross sectional area as shown in Fig. 1. Similarly, the downward force on the body's top surface is  $\rho g h_1 A$ , so that there is a net upward or so-called *buoyant force*  $\rho g A (h_2 - h_1) = \rho g A h$ . But  $A h$  is just the body's volume  $V$ , so that the buoyant force may be written as simply  $\rho g V$ . Then, since  $\rho$  is the fluid's density,  $\rho g V$  is identical with the weight of the displaced fluid. This result, namely that the buoyant force (which, being an upward force, gives rise to an apparent loss of weight of the submerged body) is equal to the weight of the displaced fluid, is known as *Archimedes' principle*.

The specific gravity of a solid heavier than water may be easily determined by the application of this principle. The body is weighed in air; then it is weighed in water, that is, suspended by a thread from the arm of a balance so as to be completely submerged (see Fig. 2). The loss of weight in water is  $W - W_1$ , where  $W$  is the weight in air and  $W_1$  is the weight in water. But the loss of weight in water is equal to the weight of the water displaced, or the weight of an equal volume of water. Thus, the specific gravity  $S$  will be

$$S = \frac{W}{W - W_1} = \frac{Mg}{Mg - M_1g} = \frac{M}{M - M_1} \quad (1)$$

where  $M$  is the body's mass in air and  $M_1$  its apparent mass in water.

\*The CGS (centimeter-gram-second) system will be used in this experiment as it is both more usual and more convenient for this work.

The specific gravity of a liquid may be found by measuring the loss of weight of a convenient solid body when immersed in that liquid and the loss of weight when immersed in water. The procedure is as follows: A heavy body is weighed in air; this weight is called  $W$ . Then it is weighed in water; this weight is called  $W_1$ . Finally, it is weighed in the liquid whose specific gravity is to be determined; this weight is called  $W_2$ . The specific gravity of the liquid will then be

$$S = \frac{W - W_2}{W - W_1} = \frac{M - M_2}{M - M_1} \quad (2)$$

since this expression represents the weight of a certain volume of the liquid divided by the weight of an equal volume of water. Here  $W - W_1$  is the loss of weight in water, and  $W - W_2$  is the loss of weight in the given liquid. In order to find the specific gravity of a solid lighter than water, it is necessary to employ an auxiliary body, or sinker, of sufficient weight and density

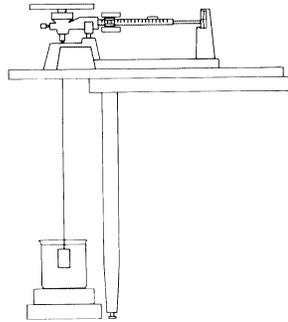


Figure 2 Arrangement for weighing a body in water

to hold the other body completely submerged. The specific gravity of a solid lighter than water, as obtained by the sinker method, is given by

$$S = \frac{W}{W_1 - W_2} = \frac{M}{M_1 - M_2} \quad (3)$$

where  $W$  is the weight of the solid in air;  $W_1$  is the weight of the solid and the sinker, with the sinker alone immersed; and  $W_2$  is the weight when both solids are immersed in water.

The hydrometer is an instrument designed to indicate the specific gravity of a liquid by the depth to which it sinks in the liquid. To measure the specific gravity of a liquid by means of a hydrometer, it is only necessary to let the hydrometer float in the liquid and to read the specific gravity directly on the calibrated scale. The reading is taken, if possible, by placing the eye below the liquid surface and seeing where this surface cuts the hydrometer scale.

## Procedure

1. Make sure the following is set up
  - Interface is plugged in and turned on.
  - Computer is connected to interface with USB cable.
  - Force sensor is plugged into the analog channel A.
2. Open data studio.
3. Click on create an experiment.
4. Click on the virtual interface analog 1 connection. Click to connect the regular force sensor, not the student force sensor.
5. Under displays, click on digits.
6. Click the tare button to zero
7. Attach metal to string.
8. Click start and find the weight of the metal in air ( $F_{\text{air}}$ ) Record.
9. Find the weight of the empty catch bucket. .

10. Fill the can with the pour spout to the spout.
11. Center can under the force sensor and the catch bucket under the spout..
12. Place the metal in the can and catch the runoff.
13. Center so the metal doesn't touch the side of the can.
14. Find the weight of the metal in the water.. Record as the weight in the water (  $F_{gwater}$  )
15. Subtract the the  $F_{gwater}$  weight in water from the  $F_{gair}$  (weight in air). This is the buoyancy  $F_b$ .
16. Find the weight of the catch bucket and water.
17. Subtract the weight of the catch bucket (step 8) from the weight of the water and catch bucket. This is the experimental buoyancy.
18. Calculate the percent error using the buoyancy from step 14 as the calculated buoyancy and the buoyancy from step 16 as the experimental buoyancy.  $\% \text{ error} = \text{experimental} - \text{calculated}/\text{calculated} * 100.$

**Find the Density of a block of metal**

19. Attach the metal to the string.
20. Click start and find the weight of the metal in air ( $F_{gair}$ ) Record.
21. Find the weight of the empty catch bucket. .
22. Fill the can with the pour spout to the spout.
23. Center can under the force sensor and the catch bucket under the spout..
24. Place the metal in the can and catch the runoff.
25. Center so the metal doesn't touch the side of the can.
26. Find the weight of the metal in the water.. Record as the weight in the water (  $F_{gwater}$  )
27. Subtract the the  $F_{gwater}$  weight in water from the  $F_{gair}$  (weight in air). This is the buoyancy  $F_b$ .
28. Find the weight of the catch bucket and water.
29. Subtract the weight of the catch bucket (step 20) from the weight of the water and catch bucket. This is the buoyancy.
30. Calculate the density of the metal.  $F_{g \text{ in air}}/F_b = \rho_{object}/ \rho_{water} .$
31. Look up the density of your metal from the chart.
32. Calculate the percent error.  $\text{Experimental} - \text{from chart}/\text{from chart} * 100$
33. Choose a second metal. And repeat the procedure.
34. Complete lab report. You must include Name, procedure, data, discussion, and conclusion.

**OBJECT 1**

	Metal in air	Metal in water	Buoyant Force (Experimental)	Catch bucket	Catch bucket + water	Water (Theoretical)
Weight (N)						

Percent error	
Density of metal	

**OBJECT 2**

	Metal in air	Metal in water	Buoyant Force (Experimental)	Catch bucket	Catch bucket + water	Water (Theoretical)
Weight (N)						

Percent error	
Density of metal	